

**LECTURE NOTES**

**ON**

**Power System Operation & Control**  
**(20A02701a)**

**IV B. Tech I Semester (R20)**



**DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING**

**GOKULA KRISHNA COLLEGE OF ENGINEERING**

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**(Approved by AICTE, New Delhi & Affiliated to JNTUA, Anantapuramu)**



**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR**  
**B.Tech (EEE)– IV-I Sem** **L T P C**  
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**(20A02701a) POWER SYSTEM OPERATION AND CONTROL**  
**(Professional Elective Course – III)**

**Course Objectives:**

- To know about economic load dispatch problems with and without losses in Power Systems
- To distinguish between hydro-electric and thermal plants and coordination between them
- To understand about optimal power flow problems and solving using specified method
- To understand about Automatic Generation Control problems and solutions in Power Systems
- To understand necessity of reactive power control, compensation under no-load and load operation of transmission systems
- To understand about deregulation aspects in Power Systems

**Course Outcomes:**

- Understand to deal with problems in Power System as Power System Engineer
- Understand to deal with AGC problems in Power System
- Analyze the problems in hydro electric and hydro thermal problems
- Evaluate the complexity of reactive power control problems and to deal with them
- Understand the necessity of deregulation aspects and demand side management problems in the modern power system era.

**UNIT IECONOMIC OPERATION OF POWER SYSTEMS**

Brief description about electrical power systems, introduction to power system operation and control, Characteristics of various steam units, combined cycle plants, cogeneration plants, Steam units economic dispatch problem with & without considering losses and its solutions, B Matrix loss formula – Numerical problems

**UNIT IIHYDRO-THERMAL COORDINATION AND OPTIMAL POWER FLOW**

**Hydro-thermal Coordination:** Characteristics of various types of hydro-electric plants and their models, Introduction to hydro-thermal Coordination, Scheduling energy with hydro-thermal coordination, Short-term hydro-thermal scheduling. **Optimal Power Flow:** Optimal power flow problem formulation for loss and cost minimisation, Solution of optimal power flow problem using Newton's method and Linear Programming technique – Numerical problems

**UNIT IIIAUTOMATIC GENERATION CONTROL**

Speed governing mechanism, modelling of speed governing mechanism, models of various types of thermal plants (first order), definitions of control area, Block diagram representation of an isolated power system, Automatic Load Frequency control of single area system with and without control, Steady state and dynamic responses of single area ALFC loop, Automatic Load-frequency control of two area system, Tie-line bias control of two area and multi-area system, Static response of two-area system – Numerical examples

**UNIT IVREACTIVE POWER CONTROL**

Requirements in ac power transmission, factors affecting stability & voltage control, fundamental transmission line equation, surge impedance, Natural loading, uncompensated line on open circuit, uncompensated line under load, types of compensations on compensated transmission lines, passive and active compensators, uniformly distributed fixed and regulated shunt compensation, series compensation, compensation by sectioning – Numerical problems

**UNIT VPOWER SYSTEMS DEREGULATION**

Principle of economics, utility functions, power exchanges, electricity market models, market power indices, ancillary services, transmission and distribution charges, principles of transmission charges, transmission pricing methods, demand-side management, regulatory framework – Numerical problems

**Textbooks:**

1. Power Generation, Operation and Control, Allen J. Wood and Bruce F. Wollenberg, John Wiley & Sons, Inc., New York, 2<sup>nd</sup> edition, 1996.
2. Power System Engineering, D P Kothari and I J Nagrath, McGraw Hill Education India Pvt.





Limited, Chennai, 3e, 2019..

**Reference Books:**

1. Electric Energy Systems Theory: An Introduction, Olle I. Elgerd, TMH Publishing Company Ltd., New Delhi, 2<sup>nd</sup> edition, 1983.
2. Reactive Power Control in Electric Systems, T J E Miller, John Wiley & Sons, New York, 1982.

**Online Learning Resources:**

1. <https://nptel.ac.in/courses/108104052>
2. <https://nptel.ac.in/courses/108101004>

## UNIT-1

The optimal system operation involved the consideration of economy of operation, system security, emissions at certain fossil-fuel plants, releases of water at hydrogeneration etc.

Economic operation is also called "Economic dispatch problem".

The economic load dispatch problem involves the solution of different problems.

- \* Unit-commitment (or) pre-dispatch problem

- \* On-line economic dispatch

→ Unit commitment problem, wherein it is required to select optimally out of the available generating sources to operate, to meet the expected load and provide a specified margin of operating reserve over a specified period of time.

→ The on-line economic dispatch wherein it is required to distribute the load among the generating units actually parallel with system in such manner as to minimize the total cost of supply the m-to-minute requirements of system.

With large ~~connection~~ interconnection of the electric networks the energy crisis in the world and continuous rise in prices, it is very essential to reduce the running charges of the electrical energy. i.e., reduce the fuel consumption for meeting a particular load demand.

→ The main aim in the economic dispatch problem is "to minimize the total cost of generating real power (production cost) at various stations while satisfying the loads and losses in transmission links".

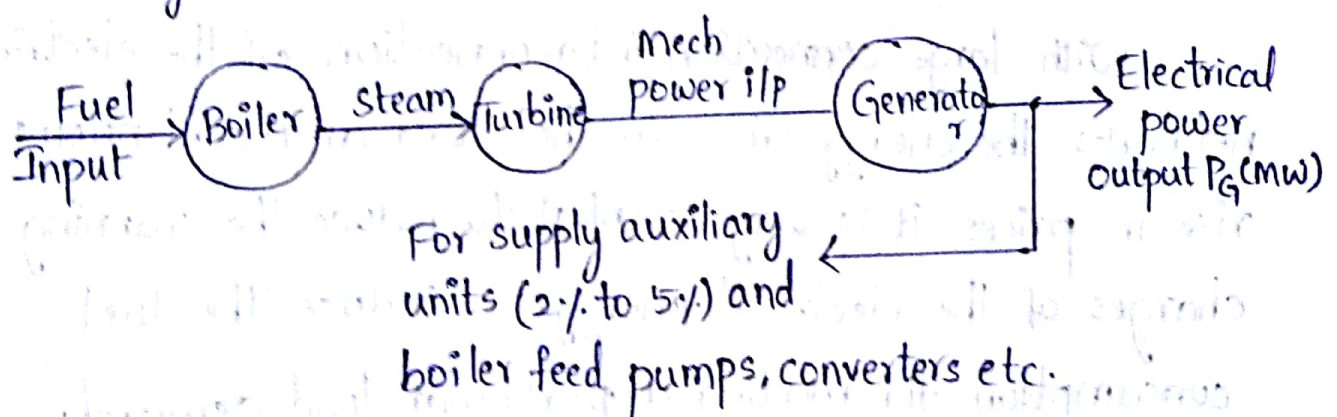
In case of economic load dispatch, the generations are not fixed but they are allowed to take values again within certain limits so as to meet a particular load demand with minimum fuel consumption.

The total cost of generation is a function of individual generation of the sources which can take values within certain constraints, the cost of generation will depend upon the system constraint for particular

In analyzing, economic operation of a thermal plant unit input-output modelling characteristics are significant.

For this function,

Consider a single unit consisting of a boiler, a turbine and a generator as shown.





This unit has to supply power not only to the load connected to power system but also the local needs for auxiliaries in the station, which may vary from 2% to 5%.

→ The power requirements for station auxiliaries are necessary to drive boiler feed pumps, fans and condenser circulating water pumps etc.,

The total input to the thermal unit could be British Thermal Unit (BTU)/hr (or) cal/hr in terms of heat supplied (or) Rs/hr in terms of the cost of fuel (coal or gas)

The total output of the unit at the generator bus will be either kW (or) MW.

System constraints:

There are two types of constraints

→ Equality constraints .... are basic load flow eqns.

→ Inequality constraints

Inequality constraints are of two types

i) Hard type ... Eg: tapping range of an on-load tap-changing t/f

ii) Soft type ... Eg: Nodal voltages and phase angles

Inequality constraints:

a) Generator constraints

$$P_{G(\min)} \leq P_G \leq P_{G(\max)}$$

Max. active power generation of a source is limited by thermal consideration and

Min. active power generation of the system is limited by flame instability of a boiler.

Similarly

Maximum and minimum reactive power generation of a source are limited.

$$Q_{G(\min)} \leq Q_G \leq Q_{G(\max)}$$

The maximum reactive power is limited because of overheating of the rotor.

The minimum reactive power is limited because of stability limit of the machine.

→ Hence the generator reactive power can't be outside the range stated by inequality.

b) Voltage constraints:

The voltage magnitudes and phase angles at various modes should vary within certain limits.

The voltage magnitudes should vary within certain limits because otherwise most of the equipments connected to the system will not operate satisfactorily (or) additional use of voltage regulating devices will make the system uneconomical.

$$|V_{P(\min)}| \leq |V_P| \leq |V_{P(\max)}|$$

$$\delta_{P(\min)} \leq \delta_P \leq \delta_{P(\max)}$$



### c) Running spare capacity constraints:

These constraints are required to meet

- i) Forced outages of one or more alternators on the system.
- ii) Unexpected load on the system.

$$G \geq P_G + P_{s0}$$

where  $P_{s0}$  is some specified power and  $G$  is the total generation.

A well planned system is one in which this spare capacity  $P_{s0}$  is minimum.

### d) Transformer tap settings:

If an auto-transformer is used, minimum tap setting could be zero and maximum be one.

$$\text{i.e., } 0 \leq t \leq 1.0$$

Similarly, for a two winding transformer if tapplings are provided on the secondary side

$$0 \leq t \leq n$$

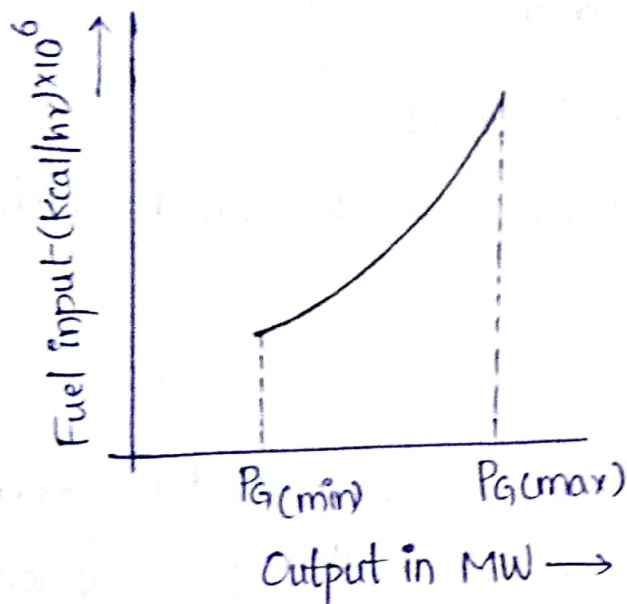
where  $n$  is the transformation ratio.

### e) Transmission line constraints:

The flow of active and reactive power through the transmission line is limited by the thermal capacity or capability of the circuit and is expressed as  $C_p \leq C_{pmax}$  where  $C_{pmax}$  is the maximum capacity loading capacity of  $p$ th line.

### f) Network security constraints

To formulate economic dispatch problem, it is necessary to know "input-output characteristics of each unit".

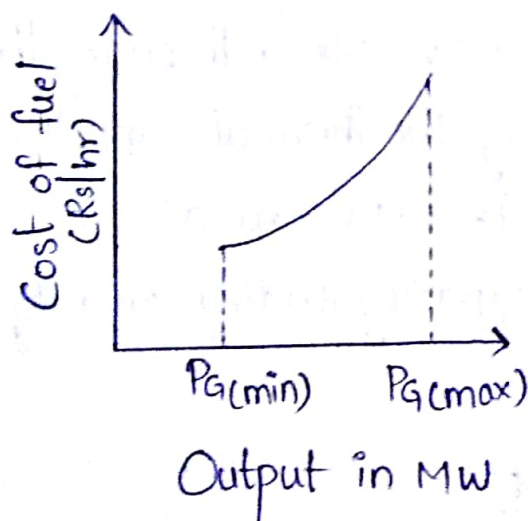


The steam turbine-generating unit curve consists of minimum and maximum limits in operation, which depend upon the steam cycle used, thermal characteristics of material, the operating temperature etc.

### Cost-curve:

To convert input-output curves into cost curves, the fuel input per hr is multiplied with cost of the fuel (expressed in Rs/million K.cal)

$$\frac{\text{K.cal} \times 10^6}{\text{hr}} \times \text{Rs/million K.cal} = \text{Rs/hr}$$



## Incremental fuel cost curve

From the input-output curves, the incremental fuel cost (IFC) curve can be obtained.

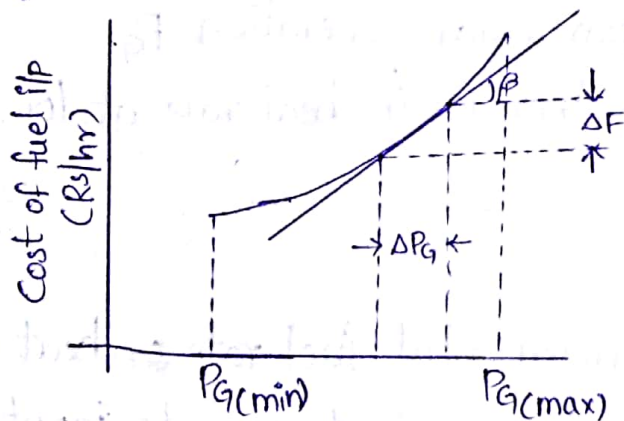
IFC is defined as "ratio of a small change in the input to the corresponding small change in the output."

$$\text{Incremental fuel cost} = \frac{\Delta \text{input}}{\Delta \text{output}} = \frac{\Delta F}{\Delta P_G}$$

where  $\Delta$  represents small changes.

As  $\Delta$  quantities become progressively smaller, it is seen that IFC is  $\frac{d(\text{input})}{d(\text{output})}$  and is expressed as Rs/MWh.

A typical plot of IFC versus output power is shown.



The IFC is now obtained as

(IC)<sub>i</sub> = slope of fuel cost curve

$$\text{temp} = \frac{\Delta F}{\Delta P_G} \text{ in Rs/MWhr}$$

$$= \lim_{P_{Gi} \rightarrow 0} \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Mathematically IFC curve expression can be obtained from the expression of cost curve.

$$C_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i$$



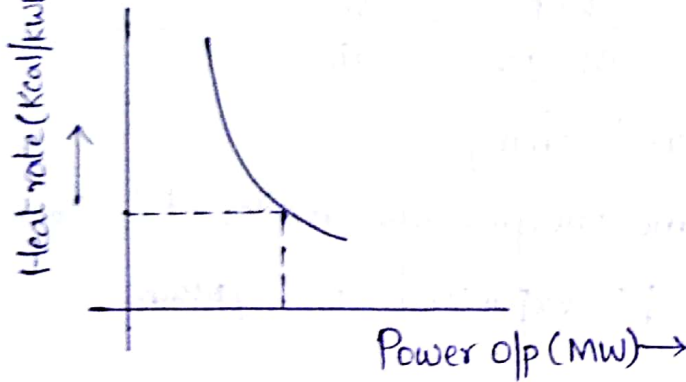
$$IFC = \frac{dc_i}{dP_{Gi}} = (IC)_i$$

$$= a_i P_{Gi} + b_i \quad \text{linear app}$$

$i = 1, 2, \dots, n$

### Heat rate curve:

The heat rate characteristic obtained from the plot of net heat rate in BTU/KWh (or) kcal/KWh vs o/p power (KW) is shown.



- The thermal unit is most efficient at minimum heat rate which corresponds to a particular generation  $P_G$ .
- The curve indicates an increase in heat rate at low and high power limits.

### Incremental efficiency:

The reciprocal of incremental fuel rate (or) heat rate which is defined as "ratio of output energy to input energy" gives a measure of full efficiency for the input.

$$\text{i.e., Incremental efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{dP_G}{dc}$$

### Optimization problem - Mathematical formulation

An optimization problem consists of

- i) Objective function
- ii) Constraint equation.

## Objective Function:

The objective function is to minimize the overall cost of production of power generation.

Cost of thermal and nuclear stations is nothing but the cost of fuel.

Let 'n' be no. of units in the system,  $C_i$  is cost of power generation of unit 'i'.

$$\begin{aligned}\text{Total cost, } C &= C_1 + C_2 + \dots + C_n \\ &= \sum_{i=1}^n C_i(P_{Gi})\end{aligned}$$

## Constraint equations:

### (1) Equality constraints:

The sum of real power generation of all the various units must always be equal to the total real-power demand on the system.

$$\text{i.e., } P_D = \sum_{i=1}^n P_{Gi} \quad (\text{or})$$

$$\sum P_{Gi} - P_D = 0$$

This is known as real power balance equation when losses are neglected.

### (2) Inequality constraints:

These constraints are considered in an economic power system operation due to physical and operational limitations of the units and components.

#### \* Output power constraints

$$P_{Gi}(\text{min}) \leq P_{Gi} \leq P_{Gi}(\text{max})$$

$$Q_{Gi}(\text{min}) \leq Q_{Gi} \leq Q_{Gi}(\text{max})$$

\* Voltage magnitude and phase angle constraints

$$V_{i(\min)} \leq V_i \leq V_{i(\max)} \text{ for } i=1,2,\dots,n$$

$$\delta_{ij(\min)} \leq \delta_{ij} \leq \delta_{ij(\max)} \text{ for } i=1,2,\dots,n$$

where  $j=1,2,\dots,m$ ,  $j \neq i$ .  $n$  is the no. of units

$m$  is no. of loads connected to each unit.

\* Dynamic constraints

$$\left| \frac{dP_{Gi}(t)}{dt} \right|_{\min} \leq \left| \frac{dP_{Gi}(t)}{dt} \right| \leq \left| \frac{dP_{Gi}(t)}{dt} \right|_{\max}$$

Similarly Q

\* Transformer tap position / setting constraints

The problem can be solved by the method Lagrange Multipliers.

$$L = \sum C_i P_{Gi} - \lambda \left[ \sum P_{Gi} - P_D \right] \quad ***$$

For minimization,  $\frac{dL}{dP_{Gi}} = 0 \Rightarrow \boxed{\frac{dC_i}{dP_{Gi}} = \lambda}$

$$\frac{dL}{dP_{Gi}} = \frac{dC_i}{dP_{Gi}} - \lambda [1-0]$$

$$\frac{dC_i}{dP_{Gi}} - \lambda = 0$$

$$\frac{dC_i}{dP_{Gi}} = \lambda ; i=1,2,\dots,k$$

where  $\frac{dC_i}{dP_{Gi}} = \text{IFC of the } i^{\text{th}} \text{ generator (Rs/MWh)}$

$$\frac{dC_1}{dP_{G1}} = \frac{dC_2}{dP_{G2}} = \dots = \frac{dC_k}{dP_{Gk}} = \lambda$$

where  $\lambda$  is the  
Lagrangian multiplier



## Economic operation of power system:

Optimum generation allocation including the effect of transmission line losses - loss coefficients - transmission line loss formula.

Consider the objective function

$$C = \sum_{i=1}^n C_i P_{Gi}$$

Subject to the following

(i) Equality constraints

$$\sum P_{Gi} = P_L + P_D \quad (\text{or}) \quad \sum P_{Gi} - P_D - P_L = 0 \longrightarrow \textcircled{1}$$

where  $P_L$  is the total transmission losses (MW)

$P_D$  is total real power demand and

$P_{Gi}$  is real power generation at the  $i^{\text{th}}$  unit

(ii) Inequality constraints.

$$P_{Gi}(\min) \leq P_{Gi} \leq P_{Gi}(\max)$$

$$Q_{Gi}(\min) \leq Q_{Gi} \leq Q_{Gi}(\max)$$

$$V_i(\min) \leq V_i \leq V_i(\max)$$

To solve the problem, we write lagrangian as

$$L = \sum C_i(P_{Gi}) - \lambda \left[ \sum_{i=1}^n P_{Gi} - P_D - P_L \right] \longrightarrow \textcircled{1}$$

$$P_L = P_L(P_{G1}, P_{G2}, \dots, P_{Gn}) \quad i=1, 2, \dots, n$$

For optimum real power dispatch,

$$\frac{dL}{dP_{Gi}} = \frac{dC_i}{dP_{Gi}} - \lambda \left[ 1 - \frac{dP_L}{dP_{Gi}} \right] = 0 \quad i=1, 2, \dots, n \longrightarrow \textcircled{2}$$

Rearranging above eqn and we get

The output of only one plant can affect the cost at only that plant, we have

$$\frac{\left(\frac{dC_i}{dP_{Gi}}\right)}{\left(1 - \frac{dP_L}{dP_{Gi}}\right)} = \lambda \quad (\text{or}) \quad \frac{dC_i}{dP_{Gi}} L_i = \lambda \quad \longrightarrow (3)$$

where  $L_i = \frac{1}{1 - \frac{dP_L}{dP_{Gi}}}$  is called penalty factor of  $i^{\text{th}}$  plant.

→ The Lagrange multiplier  $\lambda$  is in rupees per MW-hr, when fuel cost is in per hour.

From eqn(3), minimum fuel cost is obtained when the incremental fuel cost of each plant multiplied by its penalty factor is the same for all the units.

Optimum generation allocation including effect of transmission line losses:

The generating plants are generally located at longer distance from the load centre. In determination of optimum generation allocation (economic load dispatch) transmission losses should be considered. For a system consisting of 'n' no. of plants the cost of fuel input is given by

$$C_i = C_1 + C_2 + \dots + C_n$$

$$= \sum_{i=1}^n C_i(P_{Gi}) \quad [\because \text{objective function}]$$

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Subjected to constraints

$$\sum_{i=1}^n P_{Gi} = P_D + P_L$$

$$\sum_{i=1}^n P_{Gi} - P_D - P_L = 0$$

Lagrangian multiplier method is used to solve optimization problem.

By using Lagrangian we have

$$Z = \sum_{i=1}^n C_i(P_{Gi}) - \lambda \left[ \sum_{i=1}^n P_{Gi} - P_D - P_L \right] = 0$$

$$P_L = f(P_{G1}, P_{G2}, \dots, P_{Gn})$$

For optimization

$$\frac{dL}{dP_{Gi}} = 0$$

$$\frac{dL}{dP_{Gi}} = \frac{dC_i}{dP_{Gi}} - \lambda \left[ 1 - 0 - \frac{dP_L}{dP_{Gi}} \right] = 0$$

$$\frac{dC_i}{dP_{Gi}} = \lambda \left[ 1 - \frac{dP_L}{dP_{Gi}} \right]$$

$$\lambda = \frac{\frac{dC_i}{dP_{Gi}}}{1 - \frac{dP_L}{dP_{Gi}}} = \frac{IFC}{1 - ITL}$$

IFC = Incremental fuel cost

ITL = Incremental transmission loss

$$\boxed{\lambda = L_i \left( \frac{dC_i}{dP_{Gi}} \right)} \rightarrow \text{is also known as coordination equation}$$

where  $L_i$  = penalty factor

$$L_i = \frac{1}{1 - \frac{dP_L}{dP_{Gi}}}$$



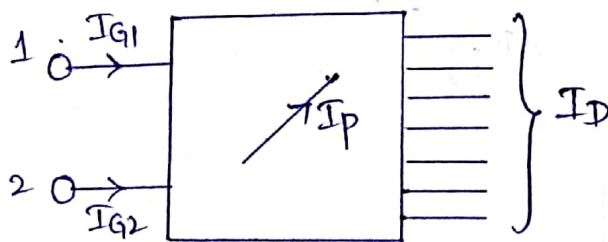
## General transmission line loss formula (Loss coefficients)

In addition to the assumptions already made, other assumptions are necessary if  $B_{ik}$  coefficients are treated to be constant. Since total load and load distribution between sources vary.

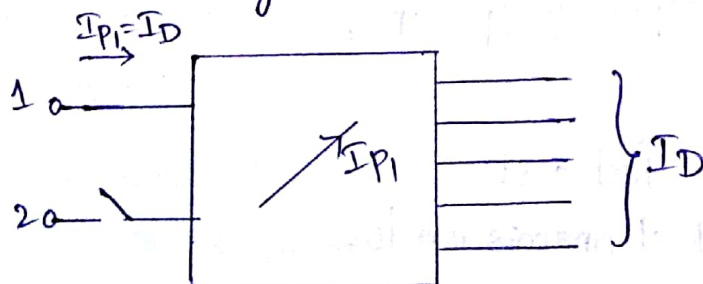
The assumptions are:

- 1) The power factor of plants remain constant.
- 2) All load currents maintain constant ratio to the total current.
- 3) Voltage magnitudes at all the plants remain constant.
- 4) Voltage phase angles at plant buses remains fixed.

Fig. shows schematic diagram showing 2 plants connected through a power system network to a no. of loads.



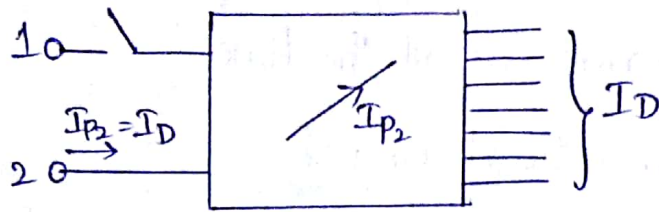
Imagine that the total current  $I_D$  is supplied by plant 1 only as shown in fig.



Let us define current distribution factors  $M_{P1}, M_{P2}$

Let the current in the line P be  $I_{P1}$

$$M_{P1} = \frac{I_{P1}}{I_D}$$



Let the current in the line P be  $I_{P2}$

Current distribution factor  $M_{P2} = \frac{I_{P2}}{I_D}$

The values of current distribution factors depend upon the impedance of lines and their interconnection and are independent of the current  $I_D$ .

When both generators 1 & 2 supplying the current into the nlw as shown in fig.

Applying superposition principle,

The current in the line P can be expressed as

$$I_P = I_{P1} + I_{P2}$$

$$I_P = M_{P1} I_{G1} + M_{P2} I_{G2} \rightarrow \textcircled{1}$$

where  $I_{G1}$  and  $I_{G2}$  are currents supplied by plants 1 & 2 respectively.

Let us make certain simplifying assumptions at this stage listed below.

i) All the load currents have same phase angle w.r.t. common reference. The load current at  $i^{\text{th}}$  bus is given by

$$I_{Di} = |I_{Di}| \angle \delta_i - \phi_i$$

where

$\delta_i$  = the phase angle of bus voltage

$\phi_i$  = lagging phase angle of the load.



$$\underline{I_{D_i}} = |I_{D_i}| \angle \theta_i$$

$\theta_i$  = same for all the load currents at all the times.

ii) Ratio  $\frac{x}{R}$  is same for all the network branches.

$$\text{Let } I_{G_1} = |I_{G_1}| \angle \sigma_1 = |I_{G_1}| (\cos \sigma_1 + j \sin \sigma_1)$$

$$I_{G_2} = |I_{G_2}| \angle \sigma_2 = |I_{G_2}| (\cos \sigma_2 + j \sin \sigma_2)$$

where  $\sigma_1$  &  $\sigma_2$  = phase angles of  $I_{G_1}$  &  $I_{G_2}$  respectively;

Sub  $I_{G_1}$  &  $I_{G_2}$  in eqn ①

$$\underline{I_p} = M_{P_1} |I_{G_1}| (\cos \sigma_1 + j \sin \sigma_1) + M_{P_2} |I_{G_2}| (\cos \sigma_2 + j \sin \sigma_2)$$

$$|I_p|^2 = [M_{P_1} |I_{G_1}| \cos \sigma_1 + M_{P_2} |I_{G_2}| \cos \sigma_2]^2 + [M_{P_1} |I_{G_1}| \sin \sigma_1 + M_{P_2} |I_{G_2}| \sin \sigma_2]^2$$

$$\boxed{\because |a+jb|^2 = a^2 + b^2}$$

$$= M_{P_1}^2 |I_{G_1}|^2 \cos^2 \sigma_1 + M_{P_2}^2 |I_{G_2}|^2 \cos^2 \sigma_2 + 2 M_{P_1} M_{P_2} |I_{G_1}| |I_{G_2}| \cos \sigma_1 \cos \sigma_2 + M_{P_1}^2 |I_{G_1}|^2 \sin^2 \sigma_1 + M_{P_2}^2 |I_{G_2}|^2 \sin^2 \sigma_2 + 2 M_{P_1} M_{P_2} |I_{G_1}| |I_{G_2}| \sin \sigma_1 \sin \sigma_2$$

$$|I_p|^2 = M_{P_1}^2 |I_{G_1}|^2 + M_{P_2}^2 |I_{G_2}|^2 + 2 M_{P_1} M_{P_2} |I_{G_1}| |I_{G_2}| \cos(\sigma_1 - \sigma_2)$$

The plant currents are given by

$$|I_{G_1}| = \frac{P_{G_1}}{\sqrt{3} |V_1| \cos \phi_1}, \quad |I_{G_2}| = \frac{P_{G_2}}{\sqrt{3} |V_2| \cos \phi_2}$$

$$\text{Power loss} = 3 |I_p|^2 R_p$$

$$= 3 R_p \left[ M_{P_1}^2 \cdot \frac{P_{G_1}^2}{3 (V_1)^2 (\cos \phi_1)^2} + M_{P_2}^2 \cdot \frac{P_{G_2}^2}{3 (V_2)^2 (\cos \phi_2)^2} + \right.$$

$$\left. 2 M_{P_1} M_{P_2} \frac{P_{G_1} P_{G_2}}{3 |V_1| |V_2| \cos \phi_1 \cos \phi_2} \cos(\sigma_1 - \sigma_2) \right]$$

$$= \frac{M_{P_1}^2 P_{G_1}^2 R_P}{|V_1|^2 (\cos \phi_1)^2} + \frac{M_{P_2}^2 P_{G_2}^2 R_P}{|V_2|^2 (\cos \phi_2)^2} + \frac{2 M_{P_1} M_{P_2} \cos(\sigma_1 - \sigma_2) P_{G_1} P_{G_2} R_P}{|V_1| |V_2| \cos \phi_1 \cos \phi_2}$$

$$P_L = B_{11} P_{G_1}^2 + B_{22} P_{G_2}^2 + 2 B_{12} P_{G_1} P_{G_2}$$

where  $B_{11} = \frac{1}{|V_1|^2 (\cos \phi_1)^2} \sum_{p=1}^n M_{P_1}^2 R_P$

$$B_{22} = \frac{1}{|V_2|^2 (\cos \phi_2)^2} \sum_{p=1}^n M_{P_2}^2 R_P$$

$$B_{12} = \left[ \frac{M_{P_1} M_{P_2} \cos(\sigma_1 - \sigma_2)}{|V_1| |V_2| (\cos \phi_1) (\cos \phi_2)} \right] R_P$$

The terms  $B_{11}$ ,  $B_{12}$  &  $B_{22}$  are called loss coefficients (or) B-coefficients.

Note: If voltages are expressed in kV, resistances are in ' $\Omega$ ' then the units of B-coefficients are  $\text{MW}^{-1}$ .

General transmission loss formula:

If the system has k plants supplying the total load through transmission lines then the transmission loss is given by

$$P_L = \sum_{m=1}^k \sum_{n=1}^k P_{Gm} B_{mn} P_{Gn}$$

If the system has 2 plants i.e.,  $k=2$

$$P_L = \sum_{m=1}^2 \sum_{n=1}^2 P_{Gm} B_{mn} P_{Gn}$$

$$= \sum_{n=1}^2 \left[ P_{G1} B_{1n} P_{Gn} + P_{G2} B_{2n} P_{Gn} \right]$$

$$= P_{G1} B_{11} P_{G1} + P_{G1} B_{12} P_{G2} + P_{G2} B_{21} P_{G1} + P_{G2} B_{22} P_{G2}$$

$$= P_{G_1}^2 B_{11} + P_{G_1} P_{G_2} B_{12} + P_{G_1} P_{G_2} B_{21} + P_{G_2}^2 B_{22}$$

$$P_L = B_{11} P_{G_1}^2 + 2 B_{12} P_{G_1} P_{G_2} + B_{22} P_{G_2}^2$$

For a system consisting of 3 generating plants the transmission loss is given by

$$P_L = \sum_{m=1}^3 \sum_{n=1}^3 P_{G_m} B_{mn} P_{G_n}$$

$$= \sum_{n=1}^3 \left[ P_{G_1} B_{1n} P_{G_n} + P_{G_2} B_{2n} P_{G_n} + P_{G_3} B_{3n} P_{G_n} \right]$$

$$= P_{G_1} B_{11} P_{G_1} + P_{G_1} B_{12} P_{G_2} + P_{G_1} B_{13} P_{G_3} + P_{G_2} B_{21} P_{G_1} + P_{G_2} B_{22} P_{G_2} + P_{G_2} B_{23} P_{G_3} + P_{G_3} B_{31} P_{G_1} + P_{G_3} B_{32} P_{G_2} + P_{G_3}^2 B_{33}$$

$$P_L = B_{11} P_{G_1}^2 + B_{22} P_{G_2}^2 + B_{33} P_{G_3}^2 + 2 B_{12} P_{G_1} P_{G_2} + 2 B_{23} P_{G_2} P_{G_3} + 2 B_{31} P_{G_3} P_{G_1}$$

Note:

1. Since the transmission lines are symmetrical, the loss coefficients  $B_{mn}$  &  $B_{nm}$  are equal

$$\text{i.e., } B_{mn} = B_{nm}$$

2.  $B_{mn}$  (or) B-coefficients are loss coefficients & can be represented in matrix form for power system analysis.

**Optimum generation allocation with line losses neglected (or)  
Economic dispatch neglecting losses:**



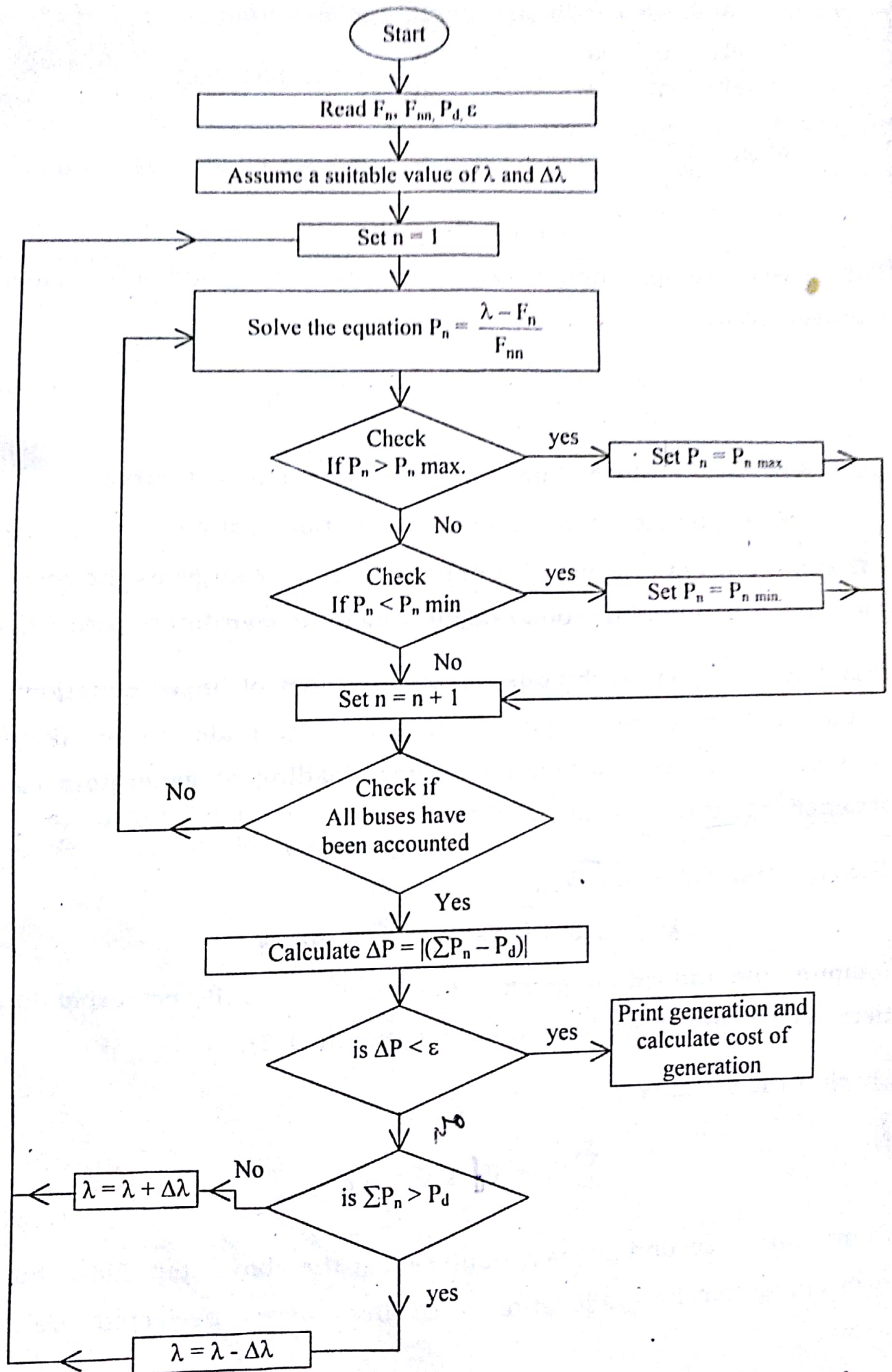


Figure (1.1) Flow chart for optimum generation allocation with line losses neglected.

**Optimum generation allocation including the effect of transmission line losses: (or)**

**Optimum load dispatch including transmission losses:**

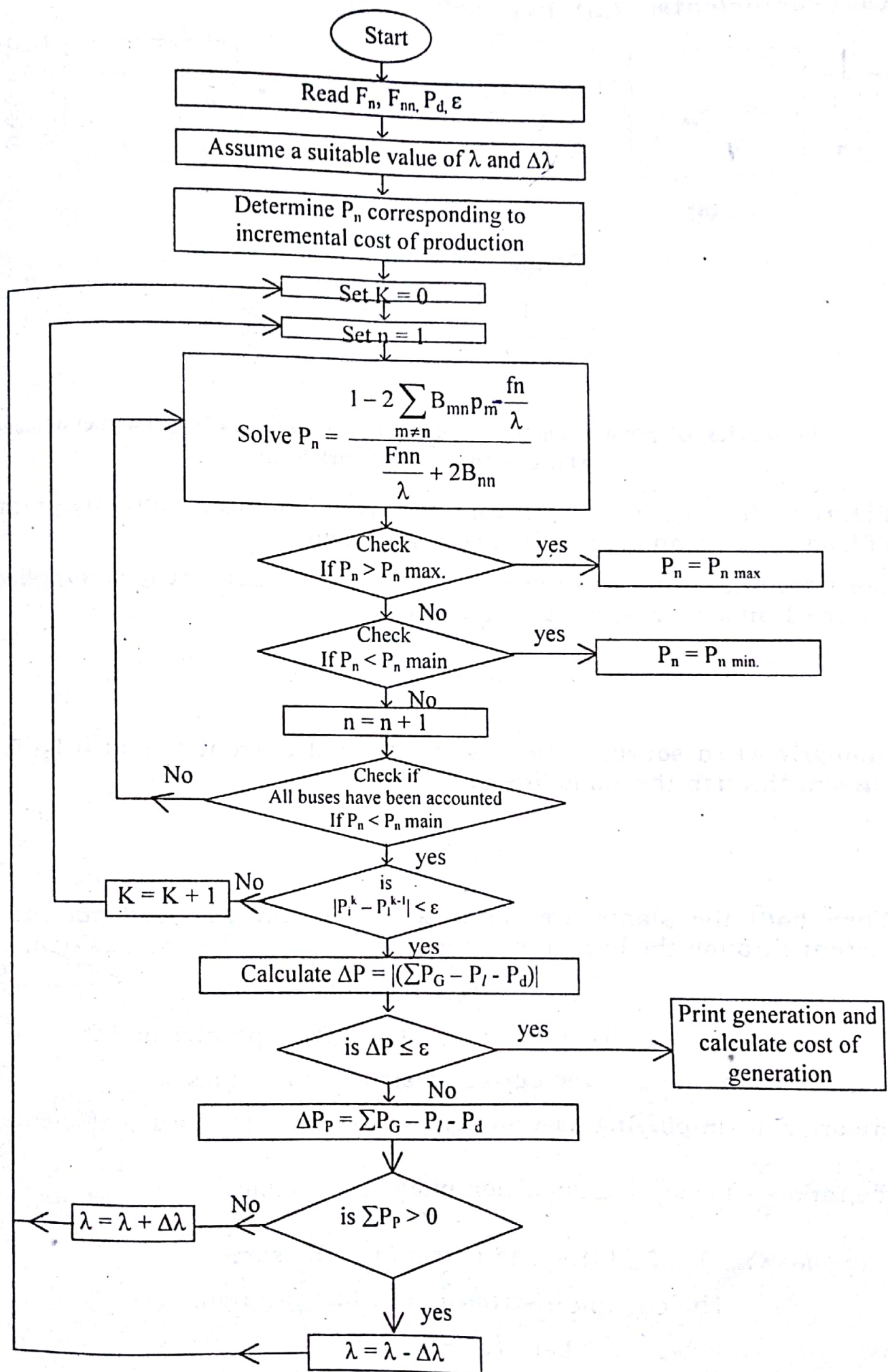


Figure (1.2) Flow chart of the optimum load dispatch including transmission losses.

## UNIT-II

### Hydrothermal Scheduling

#### Introduction :

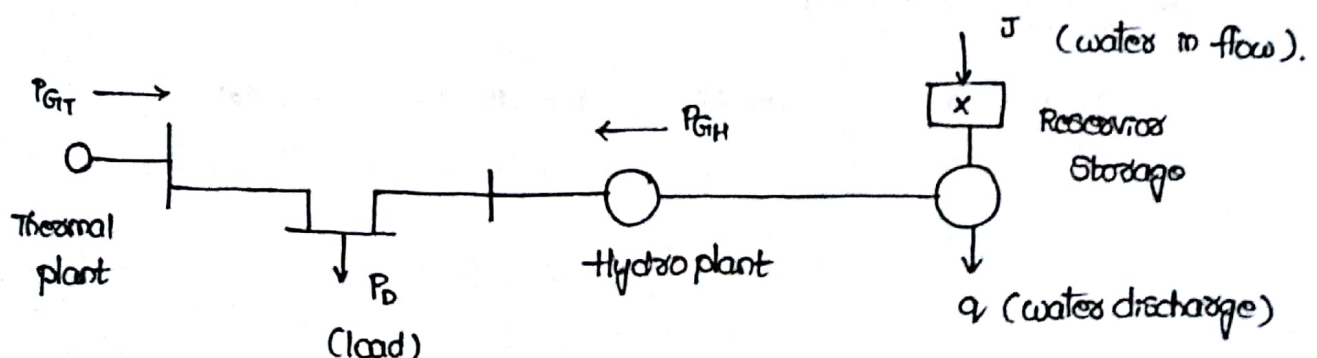
Hydro plants Can be started Easily and Can be assigned load in Very short time. However, in case of thermal plants it requires some hours to make boiler, Superheaters and turbine systems ready to take the load. For this reason hydroplants Can handle fast changing loads effectively. The thermal plants in contrast are slow in response and more suitable to Operate as base load plants. Hydroplants are Used to Operate as peak load plants.

#### Hydrothermal Scheduling :

The Operating Cost of thermal plants is Very high and at the same time its Capital Cost is low when Compared to hydroelectric plant. The Operating Cost of hydroelectric plant is low, Capital Cost is high. It is Economical as well as convenient to run both thermal as well as hydroplants in same grid.

\* The problem of minimising Operation Cost of hydrothermal system Can be viewed as minimization of Cost of thermal plants under the Constraint of the water availability for hydro generation during given period of time. This problem is known as dynamic Optimization problem where the time-factor is to be Considered.

Consider a simple hydrothermal system as shown in figure





Hydrothermal System Consists of One thermal and One Hydel plant Supplying power to the load Connected at the Centre (in between the plants & is referred as fundamental System). The Optimization will be Carried Out with real power generation as Control Variable.

### Mathematical formulation :

For a Certain period of time 'T' (1 year or) 1 month (or) 1 day).

It is assumed that

- (i). Storage of hydroreservoir at the beginning and at the Ending of the period are Specified.
- (ii). Water in flow to the reservoir and the load demand On the System are Known as function of time it Completes Certainty.

The problem is to determine water discharge rate so as to minimize the Cost of the thermal generation ' $C_T$ ' i.e

$$C_T = \int_0^T c' [P_{GT}(t)] dt \longrightarrow (1).$$

Under the following Constraints ,

- (i). Meeting the load demand

$$P_{GT}(t) + P_{GH}(t) - P_D(t) - P_L(t) = 0 \quad ; \quad t \in [0, T] \\ \longrightarrow (2).$$

This Equation (2) is Called power balance Equation.

- (ii). Water availability

$$x'(T) - x'(0) - \int_0^T \pi(t) dt + \int_0^T q(t) dt = 0 \longrightarrow (3).$$

Where ,  $x(t)$  = Water in flow

$x'(t)$  = Water storage

$x'(0), x'(T)$  = Water storage at the beginning and at the end of Optimization interval.

$q(t)$  = Water discharge.

(ii). The Hydro generation  $P_{GH}(t)$  is the function of the water discharge & storage.

$$P_{GH}(t) = f(x'(t), q(t)) \longrightarrow (4).$$

The problem Can be handled Conveniently by discretization method. The Optimization interval 'T' is Subdivided into 'm' sub intervals Each of the time length ' $\Delta T$ '. Over Each Subinterval it is assumed that all the Variables remains fixed in Value. Now the problem is represented as

$$\sum c' [P_{GT}(t)] \Delta T = \sum_{m=1}^m c(P_{GT}^m) \longrightarrow (5) \quad m=1, 2, \dots, m$$

$$q^m \quad (m=1, 2, 3, \dots, m).$$

Under the following Constraints ,

(i). power balance Equation

$$P_{GT}^m + P_{GH}^m - P_D^m - P_L^m = 0 \longrightarrow (6).$$

Where ,  $P_{GT}^m$  = Thermal Generation in the  $m^{th}$  interval.

$P_{GH}^m$  = Hydel Generation in the  $m^{th}$  interval.

$P_D^m$  = Load demand in the  $m^{th}$  interval.

$P_L^m$  = Transmission loss in the  $m^{th}$  interval.

$$P_L^m = B_{TT} (P_{GT}^m)^2 + B_{HH} (P_{GH}^m)^2 + 2 B_{TH} P_{GT}^m P_{GH}^m$$

$$[\therefore P_L = B_{11} P_{G1}^2 + B_{22} P_{G2}^2 + 2 B_{12} P_{G1} P_{G2}]$$

(ii). Water Continuity Equation

$$x'^m - x'^{(m-1)} - J^m \Delta T + q^m \Delta T = 0$$

$x'^m$  = Water storage at the end of  $m^{\text{th}}$  interval.

$J^m$  = Water in flow in the  $m^{\text{th}}$  interval.

$q^m$  = Water discharge in  $m^{\text{th}}$  interval.

The above Equation Can be written as

$$x^m - x^{m-1} - J^m + q^m = 0 \longrightarrow (*)$$

$$x^m = \frac{x'^m}{\Delta T} ; x^{m-1} = \frac{x'^{(m-1)}}{\Delta T} \quad m = 1, 2, 3, \dots, M$$

(iii). Hydro generation in any Sub-interval Can be Expressed as

$$P_{GH}^m = h_0 \{ 1 + 0.5 e (x^m + x^{m-1}) \} (q^m - p) \longrightarrow (8).$$

Where ,  $h_0 = 9.81 \times 10^3 h'_0$

$h'_0$  = Basic water head Corresponding to dead storage.

$$P_{GH}^m = 9.81 \times 10^3 h_{av}^m (q^m - p) \text{ MW}$$

Where ,  $q^m - p$  = Effective discharge

$h_{av}^m$  = average head in the  $m^{\text{th}}$  interval

$e$  = water head Correction factor

$p$  = Non-Effective discharge (water discharge needed to run hydel plant at no load).

In the above problem , formulation it is Convenient to choose water discharges in all the Sub intervals . Water discharge is in One of the Sub-intervals is a dependent Variable as shown below

$$x^m - x^0 - \sum J^m - \sum q^m = 0.$$

## Solution Technique :

The Hydro thermal problem is Solved Using non-linear programming technique in Conjunction with the first order gradient method.

\* The Lagrangian 'L' is formulated by developing the Cost function of the Equation (5) with Equality Constraints Equations (6) to (8) through Lagrangian multipliers  $\lambda_1^m, \lambda_2^m, \lambda_3^m$ . We can define Lagrangian as

$$L = \sum c(P_{GT}^m) - \lambda_1^m [P_{GT}^m + P_{GH}^m - P_D^m - P_L^m] + \lambda_2^m [x^m - x^{m-1} - J^m + q^m] + \lambda_3^m [P_{GH}^m - h_0 \{1 + 0.5 e (x^m + x^{m-1})\} (q^m - e)] \longrightarrow (9)$$

The Lagrangian multipliers are Obtained by Equating to Zero, the Partial derivatives of the Lagrangian with respect to dependent Variables results in the following Equations,

$$\frac{\partial L}{\partial P_{GT}^m} = 0 \Rightarrow \frac{d}{d P_{GT}^m} [C(P_{GT}^m)] - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GT}^m}\right) = 0 \longrightarrow (10).$$

$$\frac{\partial L}{\partial P_{GH}^m} = 0 \Rightarrow 0 - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GT}^m}\right) + \lambda_3^m = 0$$

$$\frac{\partial L}{\partial P_{GT}^m} = \lambda_3^m - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GT}^m}\right) = 0 \longrightarrow (11).$$

$$\frac{\partial L}{\partial x^m} = \lambda_2^{m+1} - \lambda_2^m - \lambda_3^m [0.5 h_0 e (q^m - e)] - \lambda_3^{m+1} [0.5 h_0 e (q^{m+1} - e)] = 0 \longrightarrow (12)$$

Substitute (7) in (11) we get,

$$\frac{\partial L}{\partial q^m} = \lambda_2^m - \lambda_3^m h_0 \{1 + 0.5 e (2x^m + J^m - 2q^m + e)\} = 0 \longrightarrow (13).$$



The dual Variables for any sub interval may be Obtained as follows :

(i). Obtain  $\lambda_1^m$  from Equation (10).

(ii). Obtain  $\lambda_2^m$  from Equation (11).

(iii). Obtain  $\lambda_3^m$  from Eq (12) and other values of  $\lambda_2^i$  from Eq (13).

$$\frac{\partial L}{\partial q^m} = 0 \Rightarrow \lambda_2^m + \lambda_3^m [-h_0(1 + 0.5e[X^m + X^{m-1}])] = 0 \longrightarrow (14).$$

The above Equation is known as gradient Vector for the Optimality the gradient Vector should be zero.

**Hydrothermal Scheduling problem :**

It Can be Categorized into two . They are :

- (i). Long term Scheduling problem
- (ii). Short term Scheduling problem.

**Long term Scheduling problem :**

In long term Scheduling problem , the Scheduling time period is Considered to be 1 year. During this period , there is a large amount of Variation in the head of water which Cannot be neglected . The dynamics of head Variations is studied from water Continuity Equation . Due to Variation in the head , determination of Co-ordination Equation becomes Complex.

The long term Scheduling problem is divided into three Categories

- (i). Multi-storage Hydro Electric System.
- (ii). Cascaded Hydro Electric System.
- (iii). Multi-chain Hydro Electric System.

- \* If the location of Hydro plants are different streams and Each hydro plant has its Own reservoir then Such hydroelectric System is Called as "multi storage hydro Electric System".
- \* If the location of hydro plants are same streams and then Such hydro electric plant is Called "Cascaded Hydroelectric system".
- \* If the location of hydro plants are at different streams and Each plant follow the same stream then Such System is "Multichain hydro electric s/m".

### Short term Scheduling problem :

In short term Scheduling problem the Scheduling time is Considered to be 1 day or 1 week. As Scheduling period is Very small, the head of the water in the reservoir Can be assumed Constant. Short term Scheduling problem is divided into 2 Categories

#### (i). Constant head hydrothermal Scheduling :

The head of water in large capacity reservoir is assumed to be Constant Over a period of the Operation.

#### (ii). Variable head hydrothermal Scheduling :

If the Capacity of the reservoir is Small than the head of water in that reservoir is Variable. The load demand should meet the total generation with losses. The Co-ordination Equation of short term Scheduling problem is Simple and Easy.

## Short term Hydrothermal Scheduling by Kirchmayer's Method :

In this method, Co-ordination Equations are derived in terms of penalty factor of both methods for obtaining the optimal Scheduling of the hydrothermal system and hence it is also known as "penalty-factor method of Solution" of short term Hydrothermal Scheduling problem.

Let  $P_{GT_i}$  = Power generation of  $i$ th thermal plant (in MW)

$P_{GH_j}$  = power generation of  $j$ th Hydel plant (in MW)

$\frac{dC_i}{dP_{GT_i}}$  = Incremental fuel Cost of  $i$ th thermal plant (in Rs/MWhr)

$W_j$  = Quantity of water Used for power generation at  $j$ th hydro plant ( $m^3/s$ ).

$\frac{dW_j}{dP_{GH_j}}$  = Incremental water rate of  $j$ th hydro plant ( $m^3/s/MW$ ).

$\frac{dP_L}{dP_{GT_i}}$  = Incremental Transmission loss of  $i$ th thermal plant

$\frac{dP_L}{dP_{GH_j}}$  = Incremental Transmission loss of  $j$ th Hydel plant.

$\lambda$  = Lagrangian multiplier

$\beta_j$  = Constant that Converts incremental water rate of Hydel plant ' $j$ ' into incremental Cost.

$n$  = No:of plants in total.

$(n - \alpha)$  = no:of Hydel plants

$\alpha$  = No:of Thermal plants.

$T$  = Time interval during which the plant Operation is Considered.

## Problem formulation :

The Objective function to minimize the Cost of generation is given by

$$\min \sum_{i=1}^{\alpha} \int_0^T C_i dt \longrightarrow (1)$$

Subject to the Equality Constraints we get

$$\sum_{i=1}^{\alpha} P_{GIT_i} + \sum_{j=\alpha+1}^n P_{GH_j} = P_D + P_L \longrightarrow (2).$$

$$\text{and } \int_0^T W_j dt = K_j \text{ for } j = \alpha+1, \alpha+2, \dots, n \longrightarrow (3).$$

Where,  $K_j$  = Amount of water in  $m^3$  utilized during the period 'T' in  $j$ th hydro-plant.

$W_j$  = Turbine discharge in the  $j$ th plant in  $m^3/s$ .

The Co-efficient ' $\lambda_j$ ' must be selected so as to Use the Specified amount of discharge of water during the Operating period. Now objective function become

$$\min C = \sum_{i=1}^{\alpha} \int_0^T C_i dt + \sum_{j=\alpha+1}^n \lambda_j K_j$$

Substitute equation (3) in the above Equation we get ,

$$\min C = \sum_{i=1}^{\alpha} \int_0^T C_i dt + \sum_{j=\alpha+1}^n \lambda_j \int_0^T W_j dt.$$



## Modelling of Turbines :

### Modelling :

The process of achieving mathematical Equivalent to the Existing physical System is known as modelling. The following Components Can be modelled to check the performance with help of Computers by means of Suitable Software packages. For Example Pspice, E-tap, E-cadd ...etc to monitor the Generator, Transformers, transmission lines, Turbines, field System, motors ... etc).

- \* In power System, both active and reactive power are never steady and they continuously change with the rising and falling load.
- \* Steam input to the turbo generators (or) water input to hydro generators must be regulated continuously to match active power demand, failing which the machine speed will vary with consequent change in frequency and it is undesirable.
- \* The maximum permissible change in frequency is  $\pm 2\%$ .
- \* The Excitation of the generators must be regulated continuously to match the reactive power demand with the reactive power generation, otherwise Voltages at various buses may go beyond the permissible limits.
- \* In modern large interconnected Systems, manual regulation is not feasible and therefore automatic generation and Voltage regulation Equipment is installed on Each generator.

### Speed governing System of Turbine :

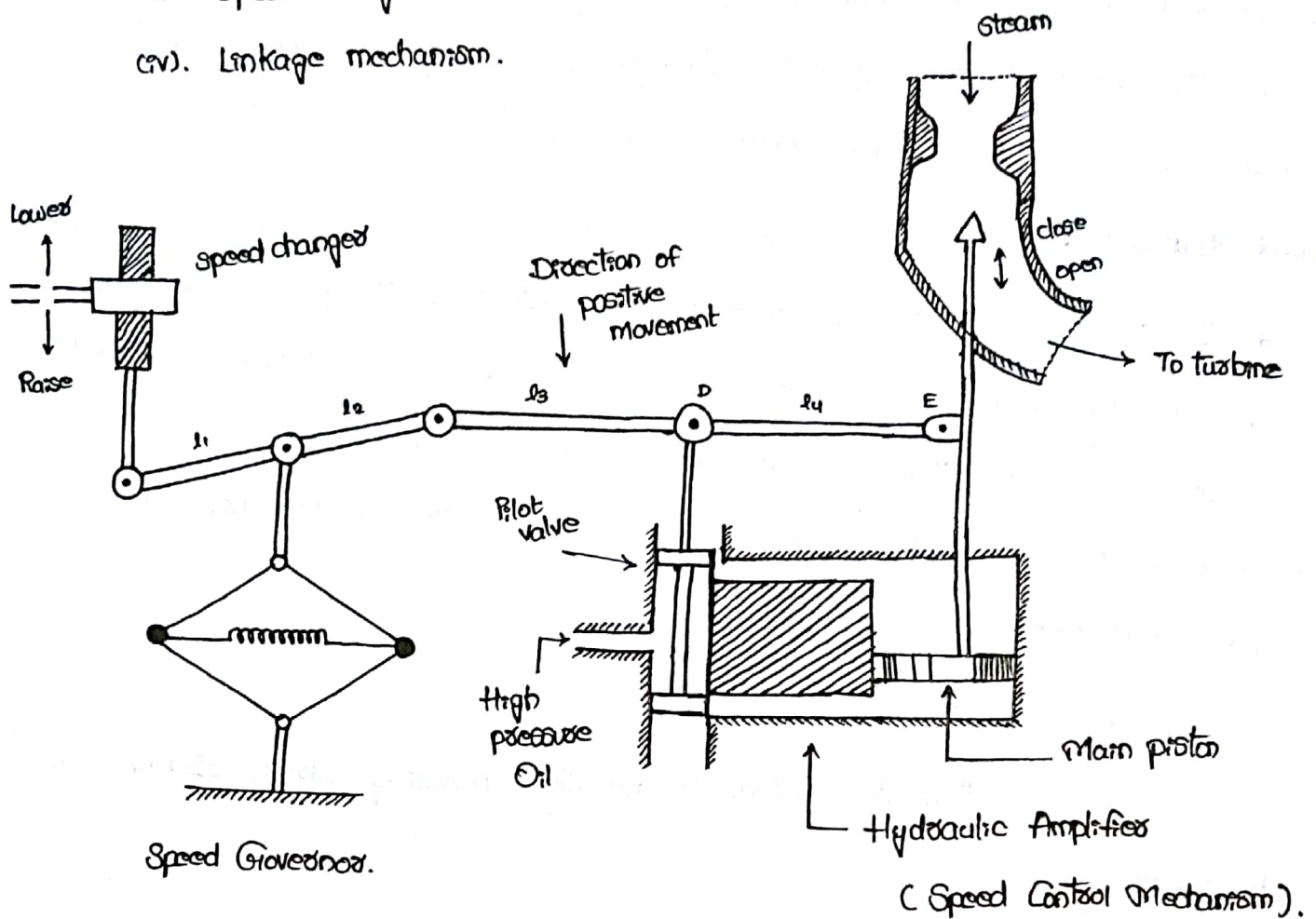
Fig shows the Schematic diagram of a Speed governing System which Controls the real power flow in the power System. The Speed governing System Consists the following parts. They are :

(i). Fly ball Speed governor

(ii). Hydraulic amplifier

(iii). Speed changer

(iv). Linkage mechanism.



### fly ball Speed governor :

\* This is the heart of the system which senses the change in the speed (frequency). With the increase in speed the fly ball move outwards and the point 'B' on linkage mechanism move downwards and vice versa.

### Hydraulic Amplifier :

- \* It comprises pilot valve and main piston arrangement.
- \* Low power level pilot valve movement is converted into high power level pilot valve movement. This is necessary in order to open or close steam valve against high pressure steam.

## Linkage Mechanism :

ABC is a rigid link pivoted at 'B' and CDE is another rigid link pivoted at 'D'. This link mechanism provides a movement to Control the Valve in proportion to change in Speed. It also provides a feedback from the Steam Valve movement.

## Speed changer :

- \* It provides a steady state power Output setting for turbine.
- \* Its downward movement Opens the upper pilot Valve. So that more steam is admitted to the turbine under steady Conditions. (Hence more steady power Output). The reverse happens for upward movement of the Speed changer.

## Modelling of Speed governor System :

Assume that the System is initially Operating Under steady State Conditions

- The linkage mechanism is stationary.
- pilot Valve is closed.
- Steam Valve is Opened by a definite magnitude.
- Turbine is running at Constant Speed.

Let  $F^0$  = System frequency

$P_G^0$  = Generation Output (or) Turbine Output  
(neglecting Generator losses).

$X_E^0$  = Steam Valve Setting Corresponding to initial Conditions.



Let the point 'A' on the linkage mechanism moved downwards by an amount  $\Delta X_A$ . It is a Command which Causes the turbine Output to change (increase in power  $\Delta P_c$  (Commanded power) and Can be written mathematically as

$$\Delta X_A = K \Delta P_c$$

\* The moment of linkage point 'A' Causes a small position changes  $\Delta X_c$  and  $\Delta X_D$  of the linkage point 'c' and 'D'.

\* If the moment of 'D' upwards by  $\Delta X_D$  high pressure Oil flows into the hydraulic amplifier from the top of main piston thereby steam Valve will move downwards by the small distance,  $\Delta X_E$  which results increase in the power.

Let Us model these Events mathematically. Two factors Contribute to the moment of 'c'. They are :

(i). Increase in frequency ' $\Delta f$ ' Causes the flyball to move Outwards. So that 'B' move downwards by  $\Delta X_B$ . This Contribution is positive and given by  $K_1 \Delta f$ .

(ii). The lowering of Speed changes by an amount  $\Delta X_A$  lifts the point 'c' upwards by an amount proportional to  $\Delta X_A$ .

\* Net moment of 'c' is given by

$$\Delta X_c = K_1 \Delta f - K_2 \Delta P_c \quad [\Delta X_A \propto \Delta P_c] \longrightarrow (1).$$

The Constants  $K_1$  and  $K_2$  depending upon length of the linkage arms AB and BC. It is proportional to the Constants of Speed changes and also the Speed governor.



\* The moment of 'D' is Contributed by moment of 'C' and 'E'. When 'D' moves upwards C & E go downwards.

∴ Net moment of 'D' is given by  $\Delta X_D = K_3 \Delta X_C + K_4 \Delta X_E \longrightarrow (2)$ .

Constants  $K_3, K_4$  depends On linkage arms CD & DE.

Assume that Volume of Oil admitted to the Cylinder is proportional to the time integral of  $\Delta X_D$ . The Value of  $\Delta X_E$  is given by

$$\Delta X_E = K_5 \int_0^t -\Delta X_D dt \longrightarrow (3).$$

Constant  $K_5$  depends On gas fluid pressure and geometry of Orifice & Cylinder

Taking laplace transform for above (1), (2), (3) Equations.

$$\Delta X_C(s) = K_1 \Delta F(s) - K_2 \Delta P_c(s) \longrightarrow (4)$$

$$\Delta X_D(s) = K_3 \Delta X_C(s) + K_4 \Delta X_E(s) \longrightarrow (5)$$

$$\Delta X_E(s) = -\frac{K_5}{s} \Delta X_D(s) \longrightarrow (6).$$

put (5) in (6), we get

$$\Delta X_E(s) = -\frac{K_5}{s} [K_3 \Delta X_C(s) + K_4 \Delta X_E(s)]$$

$$\Delta X_E(s) + \frac{K_5 K_4}{s} \Delta X_E(s) = -\frac{K_5 K_3}{s} \Delta X_C(s)$$

$$\Delta X_E(s) \left[ 1 + \frac{K_5 K_4}{s} \right] = -\frac{K_5 K_3}{s} \Delta X_C(s)$$

$$\Delta X_E(s) \left[ 1 + \frac{K_5 K_4}{s} \right] = -\frac{K_5 K_3}{s} [K_1 \Delta F(s) - K_2 \Delta P_c(s)]$$

$$\Delta X_E(s) \left[ 1 + \frac{K_5 K_4}{s} \right] = -\frac{K_1 K_3 K_5}{s} \Delta F(s) + \frac{K_2 K_3 K_5}{s} \Delta P_c(s).$$

$$\Delta X_E(s) \left[ \frac{s + K_5 K_4}{s} \right] = \left[ -\frac{K_1 K_3 K_5}{s} \Delta F(s) + \frac{K_2 K_3 K_5}{s} \Delta P_c(s) \right] \frac{1}{s}$$

$$\Delta X_E(s) \left[ \frac{s}{K_5} + K_4 \right] \cancel{K_5} = [K_3 K_2 \Delta P_c(s) - K_3 K_1 \Delta F(s)] \cancel{K_5}$$

$$\Delta X_E(s) \left[ \frac{s}{K_5} + K_4 \right] = K_3 K_2 \Delta P_c(s) - K_3 K_1 \Delta F(s)$$

$$\Delta X_E(s) = \frac{K_2 K_3 \Delta P_c(s) - K_3 K_1 \Delta F(s)}{\left[ \frac{s}{K_5} + K_4 \right]}$$

The above Equation Can also be written as

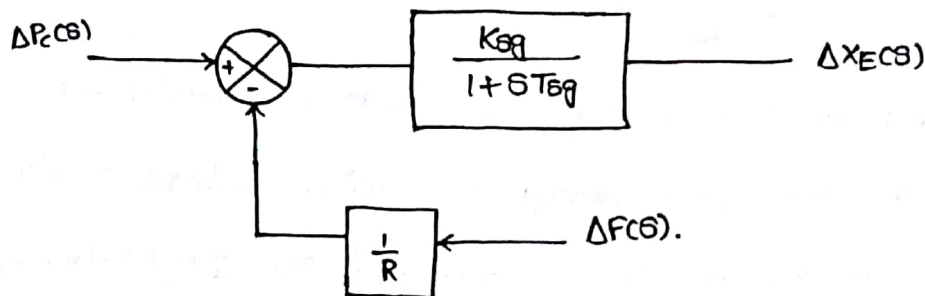
$$\Delta X_E(s) = \frac{K_{sg}}{1 + sT_{sg}} \left[ \Delta P_c(s) - \Delta F(s) \cdot \frac{1}{R} \right]$$

Where ,  $R = \frac{K_2}{K_1}$  = Speed regulation of Governor

$K_{sg} = \frac{K_2 K_3}{K_4}$  = Gain of Speed Governor

$T_{sg} = \frac{1}{K_4 K_5}$  = Time Constant of Speed Governor.

The block diagram representation of Speed governing System for the Steam turbine is given by



Note :

\* Time Constant of Speed Governor ( $T_{sg}$ ) is less than 100 ms.

## Modelling of Turbine :

The model requires relation between changes in power Output of the Steam turbine to the changes in steam Valve Opening  $\Delta X_E$ .

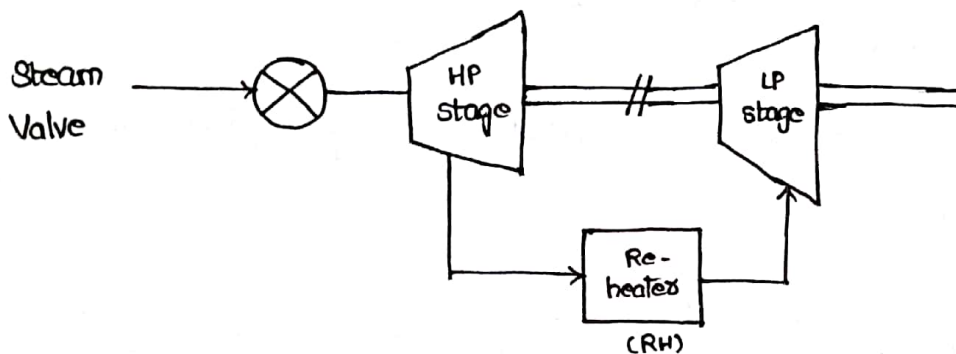
\* A steam turbine Converts stored Energy of high temperature and high pressure steam into rotating Energy which is then Converted into Electrical Energy by the generators. Generally, Steam turbines are Usually of two types

(i). Reheat type

(ii). Non-Reheat type.

### (i). Reheat Type Steam turbine :

Fig shows two stage turbine with Reheat Unit



\* The high pressure and low pressure stages are there in reheated type.

\* In Reheated type, the steam when leaving H.P. Sections returns to the boiler i.e. passed to the reheater (RH) before returning to the I.P (Intermediate pressure) section. Reheating improves efficiency of the turbine.

\* In Speed governing System, the System Valve Opens and large amount of steam enters the turbine.

\* The high pressure steam enters the HP section of the turbine. Here in the high pressure section the pressure will be dropped and temperature decreases.

To get Constant temperature, the steam from high pressure section



is given to the boiler. Here the pressure is kept constant and temperature increases to the rated value. This steam is then given to LP Section from boiler.

\* The steam from the L-P Section is dumped to Condenser, the steam is converted to water and is fed back to the boiler. Thus mechanical Output of these sections is coupled together to give rated Output Speed. The boiler is named as Reheater.

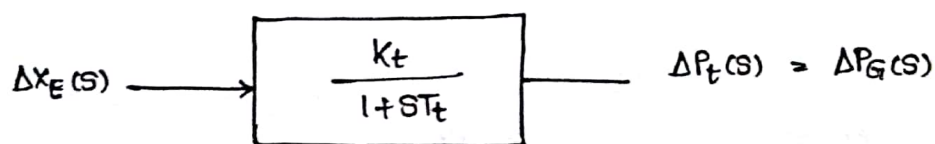
\* Reheater turbines have more than One time Constant.

\* The dynamic response of two stage turbine system is influenced by  
(i). steam which lies between the system inlet valve and 1st stage of turbine.  
(ii). Reheater storage action due to which Output of low pressure stage to lag behind that of high pressure stage.

Thus, turbine transfer function is characterised by '2' time constants  
For ease of analysis, it will be assumed that turbine can be modelled to have a single equivalent time constant.

\* Typically, time constant ' $T_t$ ' lies in the range of 0.2 to 0.5 secs.

\* Fig shows transfer function model of the steam turbine



$$\frac{\Delta P_t(s)}{\Delta X_E(s)} = \frac{K_t}{1 + sT_t}$$

Note :

\* On Opening steam Valve, the steam flow will now reach the turbine Cylinders instantaneously. The time delay experienced on this is of Order of 25 m Steam pipe.

### (iii). Non-Reheat Turbine Model :

\* In non-reheat type turbine, there is neither an Intermediate pressure section nor Reheater Unit. In this Case, the Steam is directly passed to the LP Section.

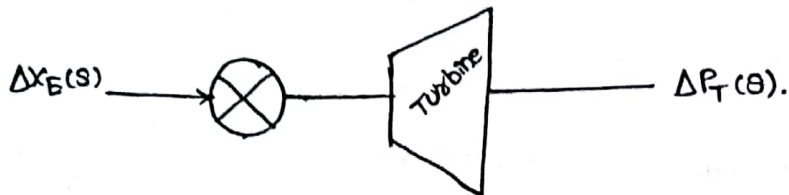
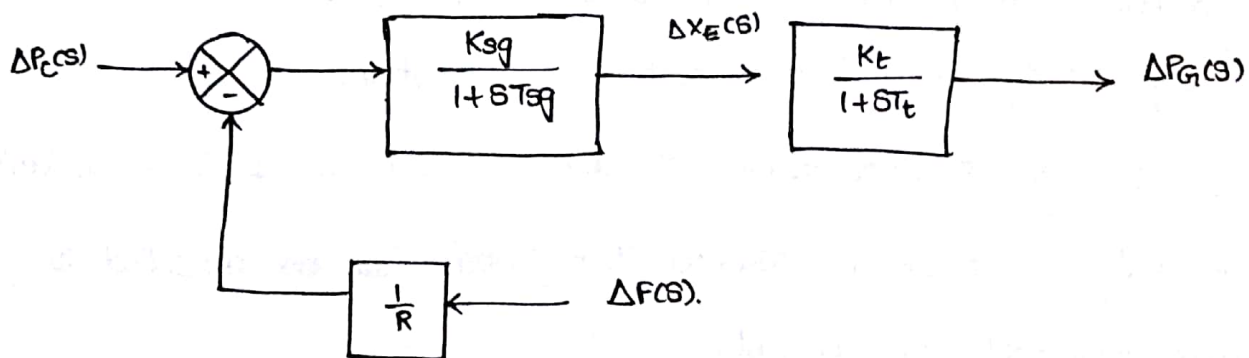


Fig. shows linearised model of non-Reheat turbine along with Speed governing mechanism.



From the figure, the Combined transfer function of turbine and Speed governing mechanism will be  $\frac{K_{sg} \cdot K_t}{(1 + sT_{sg})(1 + sT_t)}$ .

$$\therefore \frac{\Delta P_G(s)}{[\Delta P_C(s) - \frac{1}{R} \Delta P_G(s)]} = \frac{K_{sg} K_t}{(1 + sT_{sg})(1 + sT_t)}$$

## Generator Load Model :

The generator-load model gives the relation between the change in the frequency ( $\Delta f$ ) as a result of change in generation when the load changes by a small amount.

Let  $\Delta P_D$  be change in load as a result the generation also swings by an amount  $\Delta P_G$ . The net power surplus at the busbar is  $(\Delta P_G - \Delta P_D)$ . This surplus power can be absorbed by the system into two different ways. They are :

(i). By increasing the stored kinetic energy of generator rotor at a rate  $\frac{dw}{dt}$ . Where 'w' is the new value of kinetic energy.

Let 'w<sub>0</sub>' be the kinetic energy before change in load occurs at the normal speed and frequency  $f^0$ .

Let 'w' be the kinetic energy when frequency is  $f^0 + \Delta f$ .

\* Since kinetic energy (KE) is proportional to square of the speed of the governor, therefore two kinetic energies can be correlated as

$$\frac{W}{W_0} = \left( \frac{f_0 + \Delta f}{f^0} \right)^2$$

$$W = W_0 \left( 1 + \frac{2\Delta f}{f^0} + \dots \text{Higher Order terms} \right)$$

Neglecting higher order terms. Since  $\frac{\Delta f}{f^0}$  is small.

$$W = W_0 \left( 1 + \frac{2\Delta f}{f^0} \right)$$

① Differentiating above expression w.r.t 't' we get

$$\frac{dw}{dt} = \frac{2W_0}{f^0} \cdot \frac{d}{dt}(\Delta f).$$

Let  $H$  = Inertia Constant of a generator (MW-s/MVA)

$P$  = Rating of the turbo generator (MVA).



### 3. LOAD FREQUENCY CONTROL

#### Single Area Load-frequency control

Necessity of keeping frequency constant. Definitions of control area- single area control - Block diagram representation of isolated power system - steady state analysis - Dynamic response - Uncontrolled case.

In a power system, both active and reactive power demands are never steady and they continuously change with rising or falling trend.

Steam input to turbo-generators (or) water-input to hydro generators must be regulated continuously to match the active power demand, failing which consequent change in frequency and it may be highly undesirable.

The excitation of the generators must be regulated continuously to match the reactive power demand with reactive power generation, otherwise, the voltages at various system buses may go beyond the prescribed limits.

→ The maximum possible change in frequency is  $\pm 5\%$ .

#### Necessity of keeping frequency constant:

1. Loads are usually designed to operate at a particular frequency. If it is not maintained at the normal value, then frequency of operation causes electrical loads to deviate from the desired output.
2. For synchronous operation of various units in the power system network, it is necessary to maintain frequency constant.
3. Steam turbine blades are designed to operate in a narrow band of frequencies.

Deviation of frequency beyond this band may cause gradual (or) immediate turbine damage.

So, control equipment take action in case of under/over frequency.

4. Frequency affects the amount of power transmitted through interconnecting lines.

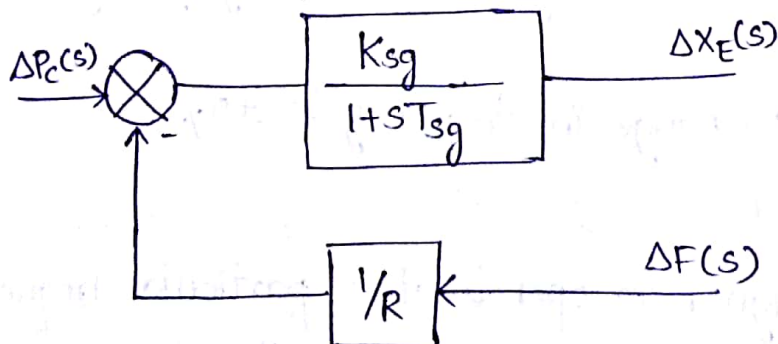
5. Electrical clocks which are driven by synchronous motors will lose or gain time in case of change in frequency.

### Block diagram representation of an isolated power system

A complete block diagram representation of an isolated power system comprising speed governor, turbine and generator-load model.

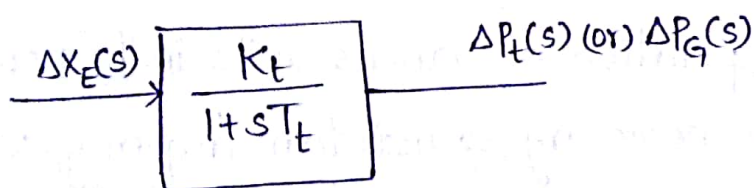
#### Speed governor model:

The block diagram of speed governor is shown.



#### Turbine model:

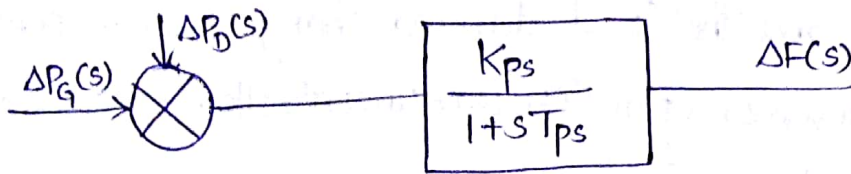
The transfer function of non-reheat type is shown:





## Generator-load model:

The transfer function of generator-load model is shown



By combining the three block diagrams i.e.,

speed-governor model,

turbine model and

generator-load model

we get complete block diagram representation of LFC.

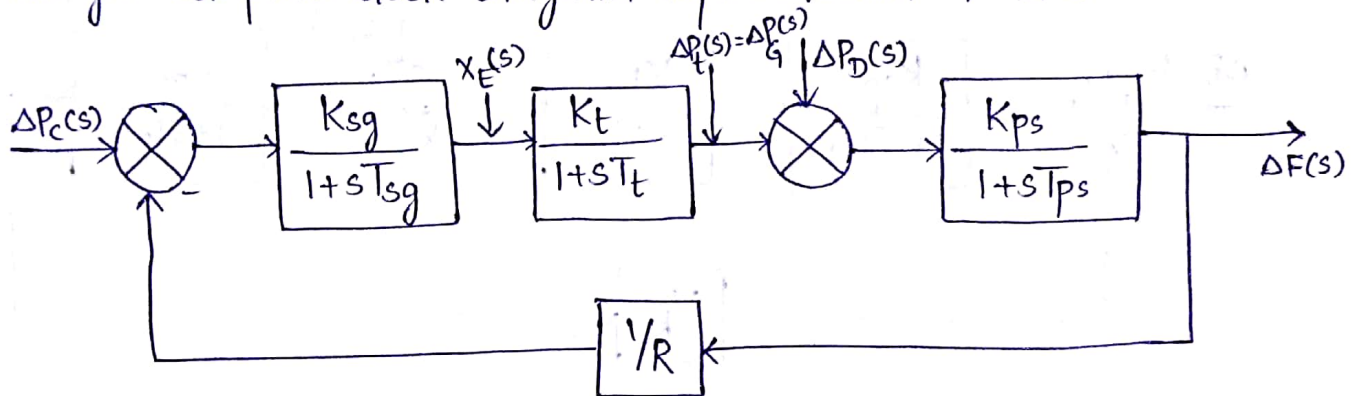


Fig. Block diagram representation of load frequency control  
(Isolated power system)

## Steady state Analysis:

There are two incremental inputs to the load frequency control system.

$\Delta P_c$  — change in speed changer setting

$\Delta P_D$  — change in load demand.

Note: Relation between  $T_{sg}$ ,  $T_t$  and  $T_{ps}$

$$T_{sg} < T_t < T_{ps}$$



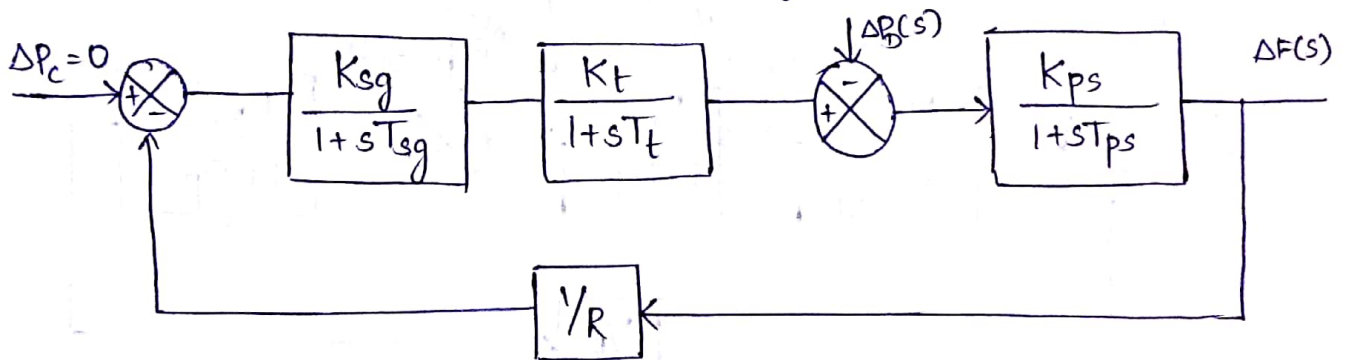
Constant speed changer position with variable load demand i.e.,  $\Delta P_c = 0$ .

Consider a case wherein speed changer has a fixed setting, which means  $\Delta P_c = 0$  and the load demand changes. Such operation is known as free governor operation (or) uncontrolled case since the speed changer is not manipulated.

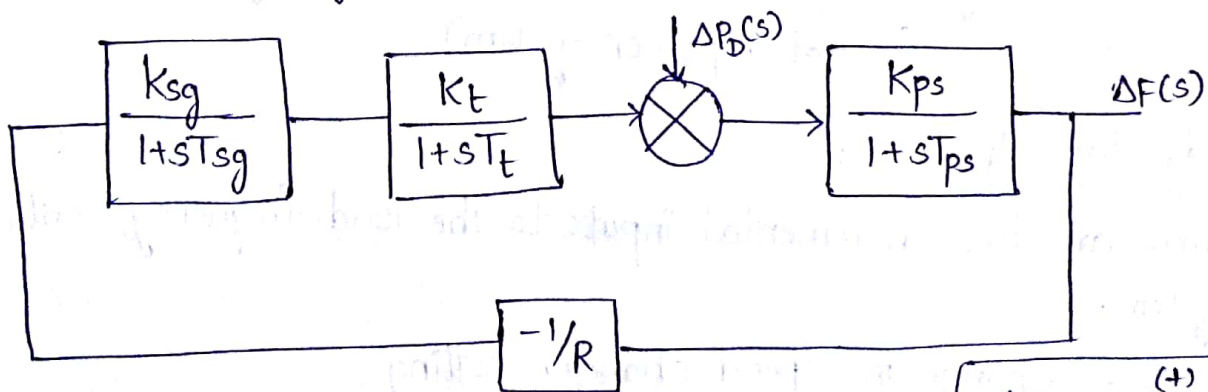
For a sudden step change of load demand

$$\Delta P_D(s) = \frac{\Delta P_D}{s}$$

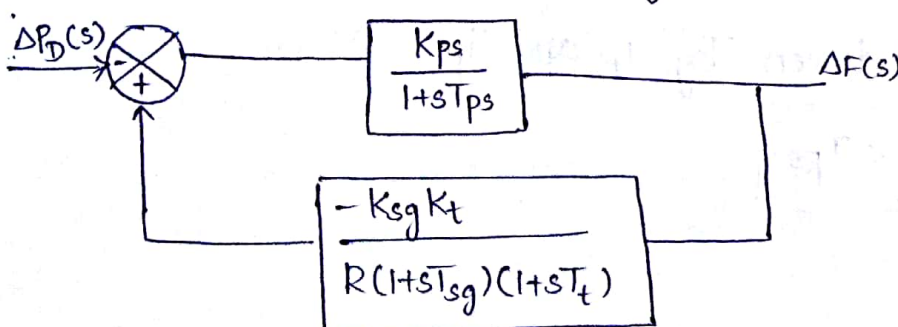
For such an operation, steady state change of frequency  $\Delta F(s)$  is to be estimated from the block diagram.



The block diagram of isolated power system now becomes..



Combining blocks in series, we get



$$\begin{aligned} \frac{\Delta F(s)}{\Delta P_D(s)} &= \frac{G(s)}{1 - G(s)H(s)} \\ &= \frac{G(s)}{1 + G(s)H(s)} \end{aligned}$$

$G(s) = \frac{K_{ps}}{1 + sT_{ps}}$   
 $H(s) = \frac{K_{sg} K_t}{R(1 + sT_{sg})(1 + sT_t)}$

$$\left. \frac{\Delta F(s)}{\Delta P_D(s)} \right|_{\Delta P_D(s)=0} = - \left[ \frac{\left( \frac{K_{ps}}{1+sT_{ps}} \right)}{1 + \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1+sT_{sg})(1+sT_t)(1+sT_{ps})} \cdot \frac{1}{R}} \right]$$

For a sudden step change of load demand

$$\Delta P_D(s) = \frac{\Delta P_D}{s} \quad \text{Multiply with } (1+sT_{ps}) \text{ both numerator and denominator}$$

$$\Delta F(s) = - \left[ \frac{K_{ps}}{(1+sT_{ps}) + \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1+sT_{sg})(1+sT_t)} \cdot \frac{1}{R}} \right] \times \frac{\Delta P_D}{s}$$

Apply final value theorem

The steady state frequency error is given by

$$\Delta f \big|_{\text{steady state}} = \lim_{s \rightarrow 0} s \cdot \Delta F(s)$$

$$\begin{aligned} \Delta f &= \lim_{s \rightarrow 0} s \left\{ - \frac{K_{ps}}{(1+sT_{ps}) + \frac{K_{sg} K_t K_{ps}}{(1+sT_{sg})(1+sT_t)} \cdot \frac{1}{R}} \right\} \cdot \frac{\Delta P_D}{s} \\ &= \left( \frac{-K_{ps}}{1 + \frac{K_{sg} K_t K_{ps}}{R}} \right) \Delta P_D \longrightarrow (2) \end{aligned}$$

where  $K_{sg}$ ,  $K_t$ ,  $K_{ps}$  are gains of speed governor, turbine and power system respectively.

The gain  $K_{sg}$  of the speed governor is easily adjustable by changing the lengths of various links of the linkage mechanism.

$K_{sg}$  is adjusted such that  $K_{sg} K_t \approx 1$

$$\Delta f = - \left( \frac{K_{ps}}{1 + \frac{K_{ps}}{R}} \right) \Delta P_D$$

From the dynamics of generator-load model.

$$K_{ps} = \frac{1}{D}, \text{ where } D = \frac{\partial P_D}{\partial f} \text{ MW/Hz}$$

$$= \left( \frac{\partial P_D}{\partial f} \right) / P_r \text{ in P.U.}$$

$$\Delta f = - \left( \frac{1/D}{1 + \frac{1}{D} \cdot \frac{1}{R}} \right) \Delta P_D$$

multiply with  $D$  both in numerator and denominator

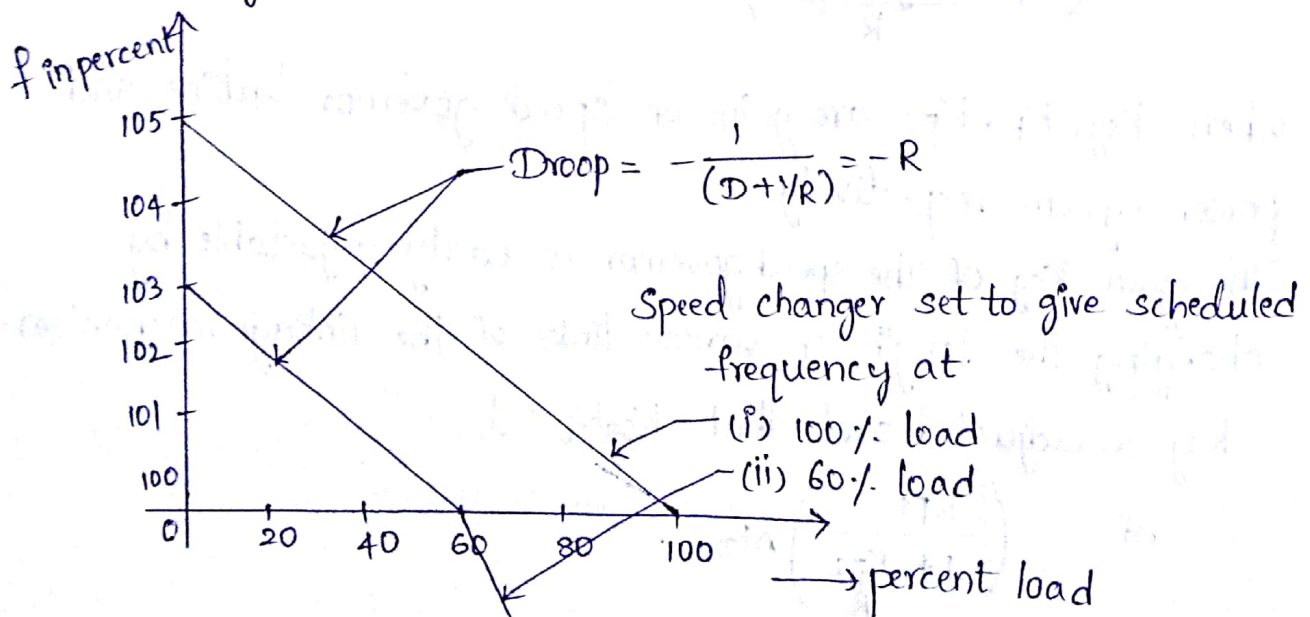
$$= - \left( \frac{1}{D + 1/R} \right) \Delta P_D$$

$$\Delta f = - \frac{1}{\beta} \Delta P_D$$

where  $\beta = (D + 1/R)$  and is known as area frequency response characteristics (AFRC) (or) area frequency regulation characteristics.

$$\Delta f = - \left( \frac{1}{D + 1/R} \right) \Delta P_D$$

The above equation gives steady state response of frequency to change in demand.





Power system parameter  $D$  is generally much smaller than  $1/R$  ( $D = 0.01 \text{ pu}$ ;  $1/R = 1/3$ ) so that  $D$  can be neglected in comparison.

Above equation is reduced to

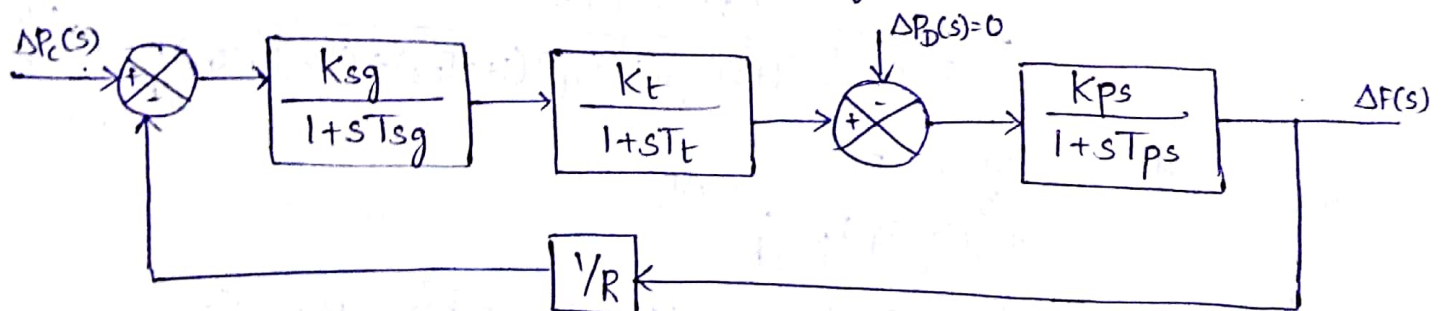
$$\Delta f = -R \cdot (\Delta P_D)$$

The droop of the load-frequency curve is mainly determined by  $R$ , the speed governor regulation.

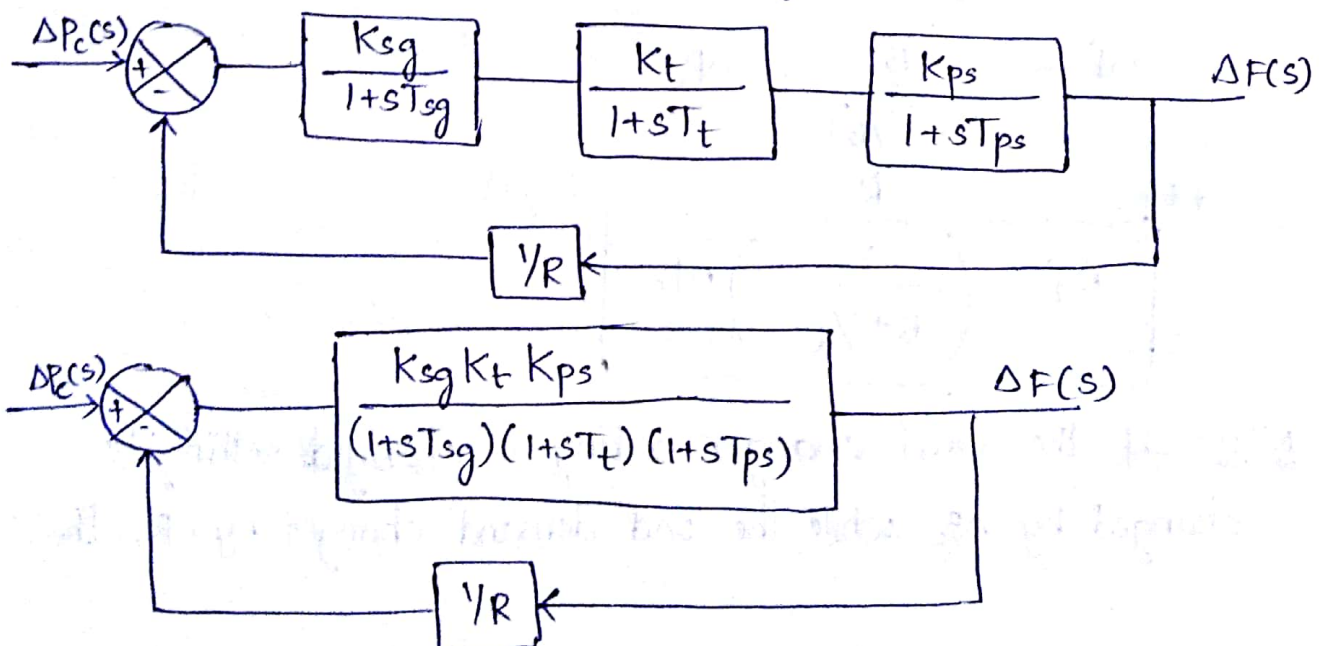
Case 2: Constant load-demand position with variable speed changer (i.e.,  $\Delta P_D = 0$ )

Consider steady effect of changing speed changer setting ( $\Delta P_c(s) = \frac{\Delta P_c}{s}$ ) with load demand remaining fixed (i.e.,  $\Delta P_D = 0$ ).

→ The steady state change in frequency is obtained as follows.



The block diagram of isolated power system now becomes,



The transfer function is given by

$$\frac{\Delta F(s)}{\Delta P_c(s)} = \frac{\left( \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1+sT_{sg})(1+sT_t)(1+sT_{ps})} \right)}{1 + \left( \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1+sT_{sg})(1+sT_t)(1+sT_{ps})} \right) \left( \frac{1}{R} \right)}$$

For a sudden step change in speed changer,  $\Delta P_c(s) = \frac{\Delta P_c}{s}$

Multiply numerator and denominator with  $(1+sT_{sg})(1+sT_t)(1+sT_{ps})$

$$\Delta F(s) = \left[ \frac{K_{sg} K_t K_{ps}}{(1+sT_{sg})(1+sT_t)(1+sT_{ps}) + \left( \frac{K_{sg} K_t K_{ps}}{R} \right)} \right] \times \frac{\Delta P_c}{s}$$

The steady state frequency error is given by

$$\begin{aligned} \Delta f \Big|_{\Delta P_D=0} &= \lim_{s \rightarrow 0} s \cdot F(s) \\ &= \lim_{s \rightarrow 0} s \left( \frac{K_{sg} K_t K_{ps}}{(1+sT_{sg})(1+sT_t)(1+sT_{ps}) + \left( \frac{K_{sg} K_t K_{ps}}{R} \right)} \right) \cdot \frac{\Delta P_c}{s} \end{aligned}$$

$$\Delta f = \frac{K_{sg} K_t K_{ps}}{1 + (K_{sg} \cdot K_t \cdot K_{ps})/R} \cdot \Delta P_c$$

Let us consider  $K_{sg} \cdot K_t \approx 1$ ,  $K_{ps} = 1/B$  where  $B = \frac{\partial P_D}{\partial f}$

The above equation becomes

$$\Delta f = \frac{\frac{1}{B}}{1 + \frac{(1/B)}{R}} \cdot \Delta P_c$$

\*\*\*

$$\boxed{\Delta f = \left( \frac{1}{B + 1/R} \right) \Delta P_c}$$

Note: If the speed changer setting is changed setting is changed by  $\Delta P_c$  while the load demand changes by  $\Delta P_D$ , the

steady frequency change is obtained by superposition.

$$\text{i.e., } \Delta f = \left( \frac{1}{B + 1/R} \right) (\Delta P_C - \Delta P_D)$$

Dynamic Response - (Uncontrolled case i.e.,  $\Delta P_C = 0$ )

The dynamic response is how fast the frequency changes as a function of time immediately after disturbance before it reaches the new steady-state condition.

Comparing the relative values of time constants, we reduce the third order LFC model to first order

For a practical LFC system

$$T_{sg} < T_t \ll T_{ps}$$

Typical values are:  $T_{sg} = 0.4s$ ;  $T_t = 0.5s$ ,  $T_{ps} = 20s$

If  $T_{sg}$  and  $T_t$  are considered negligible compared to  $T_{ps}$  and by adjusting  $K_{sg}K_t \approx 1$ , the block diagram of LFC of the power system of an isolated system is reduced to first order system as shown

For uncontrolled case,  $\Delta P_C(s) = 0$

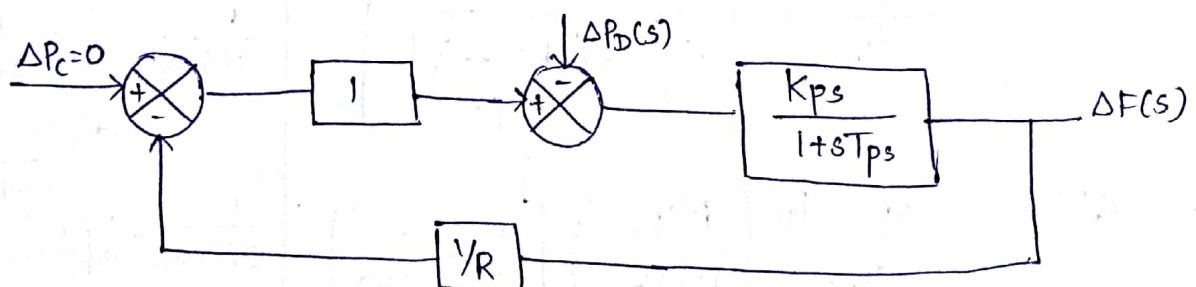
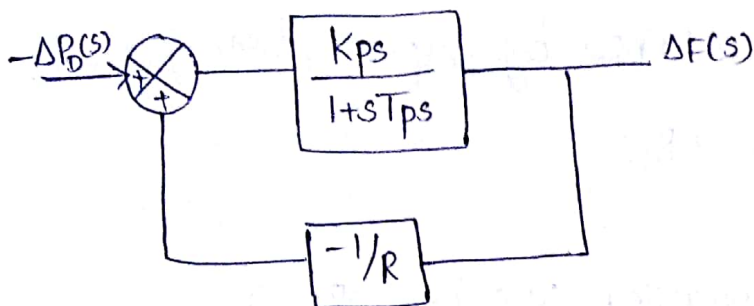


Fig. First order approximate block diagram of LFC of an isolated area





$$\frac{\Delta F(s)}{-\Delta P_D(s)} = \frac{\frac{Kps}{1+sTps}}{1 - \left(\frac{Kps}{1+sTps}\right)\left(\frac{-1}{R}\right)} = \frac{\frac{Kps}{1+sTps}}{1 + \left(\frac{Kps}{1+sTps}\right)\left(\frac{1}{R}\right)}$$

Multiplying both numerator and denominator by  $(1+sTps)$

$$\Delta F(s) = \left[ \frac{Kps}{(1+sTps) + \frac{Kps}{R}} \right] - \frac{\Delta P_D}{s}$$

$$= \left[ \frac{-Kps}{sTps + \left(\frac{R+Kps}{R}\right)} \right] \frac{\Delta P_D}{s}$$

Dividing by  $Tps$

$$= \left[ \frac{\left(\frac{-Kps}{Tps}\right)}{s + \left(\frac{R+Kps}{RTps}\right)} \right] \frac{\Delta P_D}{s}$$

$$= \frac{-Kps \cdot \Delta P_D}{Tps} \left[ \frac{1}{s \left\{ s + \left(\frac{R+Kps}{RTps}\right) \right\}} \right]$$

$$\Delta F(s) = \frac{-Kps \cdot \Delta P_D}{Tps} \left( \frac{RTps}{R+Kps} \right) \left[ \frac{1}{s} - \frac{1}{s+a} \right]$$

$$\begin{aligned} \frac{1}{s(s+a)} &= \frac{A}{s} + \frac{B}{s+a} \\ &= \frac{1/a}{s} + \frac{-1/a}{s+a} \\ \frac{1}{s(s+a)} &= \frac{1}{a} \left[ \frac{1}{s} - \frac{1}{s+a} \right] \end{aligned}$$

By taking inverse laplace

$$\begin{aligned} \Delta f(t) &= L^{-1} [\Delta F(s)] \\ &= L^{-1} \left[ \frac{-R Kps \cdot \Delta P_D}{R+Kps} \cdot \left( \frac{1}{s} - \frac{1}{s+a} \right) \right] \end{aligned}$$

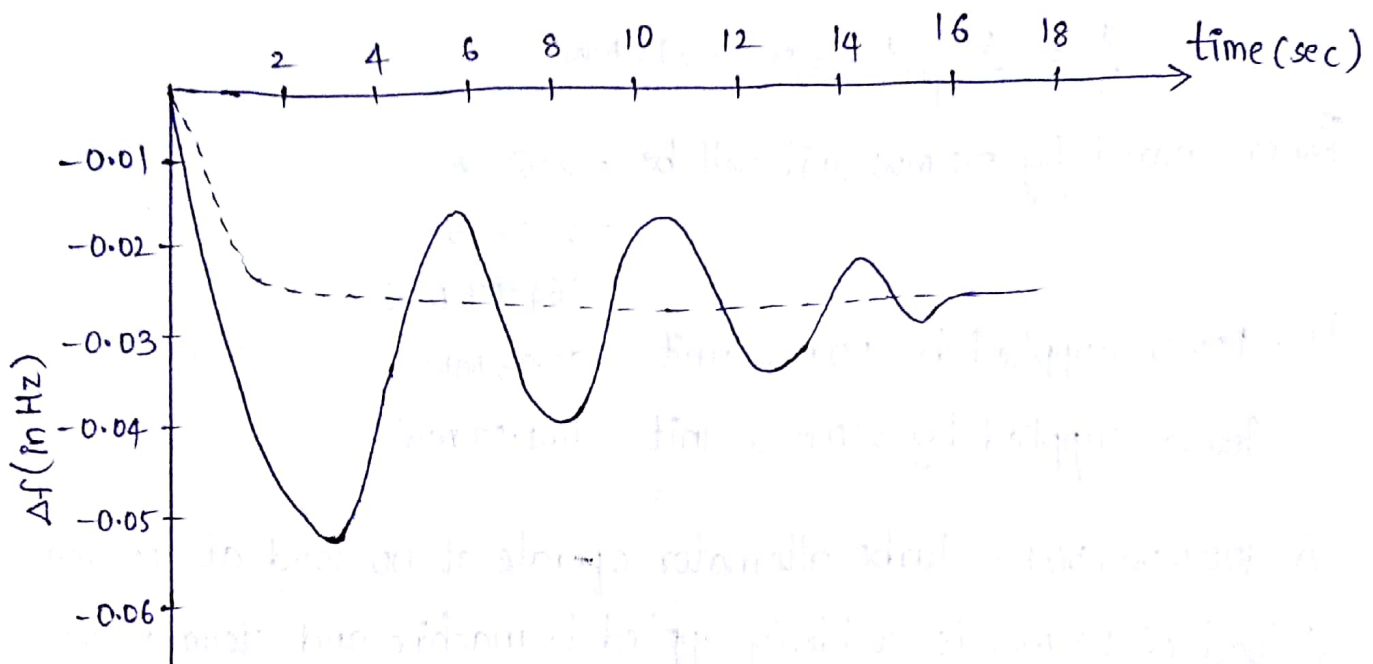
$$\Delta f(t) = \frac{-R \cdot K_{ps}}{R + K_{ps}} [1 - e^{-at}] \cdot \Delta P_D$$

$$\text{where } a = \frac{R + K_{ps}}{R T_{ps}}$$

\*\*\*

$$\Delta f(t) = \frac{-R K_{ps}}{R + K_{ps}} \left[ 1 - e^{-\frac{t}{T_{ps}} \left( \frac{R + K_{ps}}{R} \right)} \right] \Delta P_D$$

The dynamic response of change in frequency for a step-change in load is as follows:



- ① Two turbo alternators rated for 110 MW and 210 MW have governor droop characteristics of 5% from no-load to full-load. They are connected in parallel to share a load of 250 MW. Determine the load shared by each machine. Assuming free governor action.

Sol:- Since the two machines are working in parallel, the percent drop in frequency from the machines due to different loadings must be same.

Let load on generator 1 of 110 MW unit =  $x$

Load on generator 2 of 210 MW unit =  $(250 - x)$

$$\text{Percent drop in speed} = \frac{5}{110} x \longrightarrow \textcircled{1}$$

$$\text{Percent drop in speed} = \frac{5}{210} (250 - x) \longrightarrow \textcircled{2}$$

As percent drop in frequency must be same

From eqns  $\textcircled{1}$  &  $\textcircled{2}$

$$\frac{5x}{110} = \frac{5}{210} (250 - x)$$

$$\frac{x}{11} = \frac{250 - x}{21} \Rightarrow x = 85.93 \text{ MW}$$

Power shared by 210 MW unit will be =  $250 - x$

$$= 250 - 85.93$$

$$= 164.07 \text{ MW}$$

i.e., power supplied by 110 MW unit = 85.93 MW

Power supplied by 210 MW unit = 164.07 MW

- $\textcircled{2}$  A 100 MVA, 50 Hz turbo alternator operate at no-load at 3000 rpm. A load of 25 MW is suddenly applied to machine and steam valve to the turbine commence to open after 0.6 sec due to time-lag in the governor system. Assuming inertia constant H of 4.5 MW-sec/kVA of generator capacity, calculate the frequency to which the generated voltage droops before the steam flow commences to increase to meet the new load.

Sol:- By definition,  $H = \frac{\text{Stored energy (K.E.)}}{\text{Capacity of machine}}$

The energy stored at no-load =  $4.5 \times 100 = 450 \text{ MJ} = W^0$

$W^0 = \text{Inertia constant} \times \text{rating of turbo generator}$

Before the steam valve open,



The energy lost by the rotor  $= 25 \times 0.6 = 15 \text{ MJ} = W$

As a result of this, there is a reduction in speed of the rotor and therefore reduction in frequency.

$$\left(\frac{f}{f^0}\right)^2 = \left(\frac{W}{W^0}\right)$$

$W$  = new value

$W^0$  = old value

$$f = f^0 \sqrt{\left(\frac{W}{W^0}\right)}$$

$$(f^0)^2 \rightarrow W^0$$

$$(f)^2 \rightarrow W$$

$$= 50 \times \sqrt{\frac{(450-15)}{450}}$$

$$= 49.16 \text{ Hz}$$

$$\text{Frequency deviation} = \frac{50 - 49.16}{50} \times 100$$

$$= 1.68$$

- ③ Two generators rated 200 MW and 400 MW are operating in parallel. The drop characteristics of their governors are 4% and 5% respectively from no-load to full-load. Assuming that the generators are operating at 50 Hz at no-load. How would a load of 600 MW be shared between them? What will be the system frequency at this load? Assume free governor operation.

Sol:- Since the generators are in parallel, they will operate at the same frequency at steady load.

Let load on generator 1 of 200 MW =  $x$  MW

and load on generator 2 of 400 MW =  $(600 - x)$  MW

Let

Reduction in frequency =  $\Delta f$

Reduction in frequency of 200 MW will be  $\left(\frac{4}{200}\right)x$

Reduction in frequency of 400 MW will be  $\left(\frac{5}{400}\right)(600-x)$

Equating above eqns, we get

$$x = 231 \text{ MW (load on gen 1)}$$

$$600 - x = 369 \text{ MW (load on gen 2)}$$

$$\text{System frequency} = 50 - \frac{0.04 \times 50}{200} \times 231$$

$$= 47.69 \text{ Hz}$$

- ④ A 100 MVA synchronous generator operates on full-load at frequency of 50 Hz. The load is suddenly reduced to 50 MW. Due to time lag in governor system, the steam valve begins to close after 0.4 sec. Determine the change in frequency that occurs in this time. Assume  $H = 5 \text{ KW-sec/KVA}$

Sol:-

Given,

$H = 5 \text{ KW-sec/KVA}$  of generator capacity

K.E. stored in rotating parts of generator and turbine.  
(rotor)

$$K.E. = H \cdot P.$$

$$= 5 \text{ KW-sec/KVA} \cdot 100 \times 10^3 \text{ KVA}$$

$$= 500 \times 10^3 \text{ K.W-sec} = 500 \text{ MJ}$$

$$\text{Energy gained by the rotor} = 50 \text{ MW} \cdot 0.4 \text{ sec}$$

$$= 20 \text{ MW-sec} = 20 \text{ MJ}$$

$$\text{Stored K.E.} \propto (\text{frequency})^2$$

$$f = f^0 \sqrt{\frac{W}{W_0}} = 50 \sqrt{\frac{500+20}{500}}$$

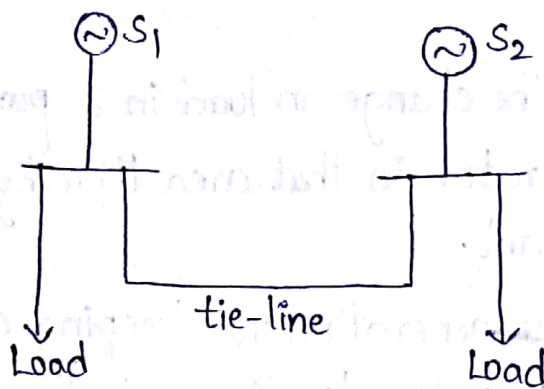
$$f = 51 \text{ Hz}$$

## Two-area load frequency control:

Load frequency control of 2-area system - Uncontrolled case and controlled case, tie-line bias control.

### Tie-line bias control:

If the system consists of a single machine connected to a group of loads the speed and frequency change in accordance with the governor characteristic as the load change. If system consists of two machines running in parallel as shown:



The possibility of sharing the load by two machines is as follows:

Tie-line: The line which connects two control areas (or two generators, here) is called tie-line.

If the change in load is either  $S_1$  or  $S_2$ , but the generation of  $S_1$  alone is regulated to adjust this change so as to have constant frequency. The method of regulation is known as "flat frequency regulation". Under such conditions  $S_2$  is said to be operating at base load.

The major drawback of flat frequency regulation is that  $S_1$



must absorb all load changes for the entire system. Therefore the tie-line between two stations would have to absorb all load changes.

Since the generator at  $S_2$  would maintain its output constant.

(The operation of generation  $S_2$  on base load has the advantages when  $S_2$  is much more efficient than the other station and it is desirable to obtain maximum output).

→ The other possibility of sharing the change in load is both  $S_1$  and  $S_2$  would <sup>regulate</sup> ~~generate~~ their generators to maintain the frequency constant. This is known as "parallel-frequency regulation".

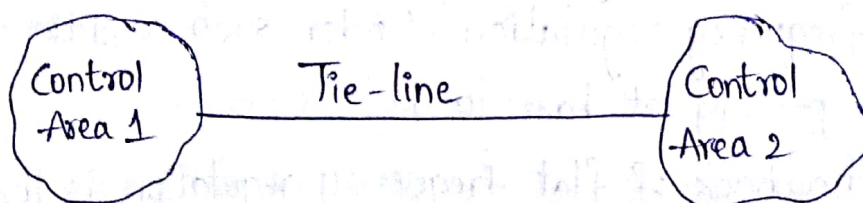
→ The third possibility is that the change in load in a particular area is taken care by the generator in that area thereby tie-line loading remains constant.

This method of regulating the generation for keeping constant frequency is known as "Flat tie-line loading control".

### Load frequency control of two-area system:

An extended power system can be divided into a number of load frequency control (LFC) areas which are interconnected by tie-lines. Such an operation is called "pool operation".

Consider two control areas connected by a single tie-line as shown.



9

The control objective is to regulate the frequency of each area and simultaneously regulate the tie-line power as per inter-area power agreement.

In case of an isolated control area, proportional plus Integral Controller (PI) will be installed so as to give zero steady state error (i.e.,  $\Delta f_{ss} = 0$ )

In two-area control case, PI controller will be installed to give zero steady state error in tie-line power (i.e.,  $\Delta P_{tie} = 0$ ) in addition to PI controller to give zero steady state error in frequency.

For the sake of convenience, each control area can be represented by an equivalent governor, turbine and generator system.

### Mathematical Treatment:

In an isolated control area case,

The incremental power ( $\Delta P_G - \Delta P_D$ ) is accounted for the rate of increase of stored kinetic energy and increase in area load caused by increase in frequency.

Since a tie line transports power in or out of an area, this fact must be ~~accounted for~~ accounted for the incremental power balance equation of each area.

→ Power transported out of area 1 is given by

$$P_{tie,1} = \frac{|V_1||V_2|}{X_{12}} \sin(\delta_1^\circ - \delta_2^\circ) \longrightarrow \textcircled{1}$$

where  $\delta_1^\circ, \delta_2^\circ$  = power angles of equivalent machines of two areas.

$X_{12}$  = tie-line reactance b/w control areas 1 & 2.



→ If there is change in load demands of two areas, there will be incremental changes in power angles ( $\Delta\delta_1, \Delta\delta_2$ ) then change in incremental tie-line power.

$$\Delta P_{\text{tie},1} = \frac{|V_1||V_2|}{X_{12}} \cos(\delta_1^\circ - \delta_2^\circ) (\Delta\delta_1 - \Delta\delta_2)$$

$$\Delta P_{\text{tie},1} = \frac{|V_1||V_2|}{P_r X_{12}} \cos(\delta_1^\circ - \delta_2^\circ) (\Delta\delta_1 - \Delta\delta_2)$$

where  $P_r$  = rating of the turbo generator (num)

$$\Delta P_{\text{tie},1} = T_{12} (\Delta\delta_1 - \Delta\delta_2)$$

$$\text{where } T_{12} = \frac{|V_1||V_2|}{P_r X_{12}} \cos(\delta_1^\circ - \delta_2^\circ)$$

= synchronizing coefficient.

Since incremental power angles are integrals of incremental frequencies we can write above eqn as

$$\Delta P_{\text{tie},1} = T_{12} (\int \omega_1 dt - \int \omega_2 dt)$$

$$\boxed{\Delta P_{\text{tie},1} = 2\pi T_{12} (\int \Delta f_1 dt - \int \Delta f_2 dt)} \longrightarrow (2)$$

where  $\Delta f_1$  and  $\Delta f_2$  are incremental changes of areas 1 and 2 respectively.

Similarly,

→ The incremental tie-line power out of area 2 is given by

$$\Delta P_{\text{tie},2} = 2\pi T_{21} (\int \Delta f_2 dt - \int \Delta f_1 dt)$$

where

$$T_{21} = \frac{|V_2||V_1|}{P_r X_{21}} \cos(\delta_2^\circ - \delta_1^\circ) = \frac{P_r T_{12}}{P_r} = a_{12} T_{12}$$



$$\Delta P_{tie,2} = -2\pi a_{12} T_{12} \left( \int \Delta f_1 dt - \int \Delta f_2 dt \right) \rightarrow (3)$$

→ The incremental power balance equation for area 1 can be written as

$$\Delta P_{G1} - \Delta P_{D1} = \frac{2H}{f_1^0} \frac{d}{dt} (\Delta f_1) + B \Delta f_1 + \Delta P_{tie,1} \rightarrow (4)$$

$$\rightarrow D \text{ (or) } B \left[ \frac{\partial P_D}{\partial f} \right] \Delta f$$

$D$  is positive for a predominately motor load.

All quantities other than frequency are in p.u.

Taking Laplace transform and reorganizing, we get

$$\Delta P_{G1}(s) - \Delta P_{D1}(s) = \frac{2H}{f_1^0} s \Delta F_1(s) + B_1 \Delta F_1(s) + \Delta P_{tie,1}(s)$$

$$\Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{tie,1}(s) = \left( \frac{2H}{f_1^0} s + B_1 \right) \Delta F_1(s)$$

$$\Delta F_1(s) = \frac{\Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{tie,1}(s)}{\left( \frac{2H}{f_1^0} s + B \right)}$$

$$\therefore B \left( s \cdot \frac{2H}{B f_1^0} + 1 \right)$$

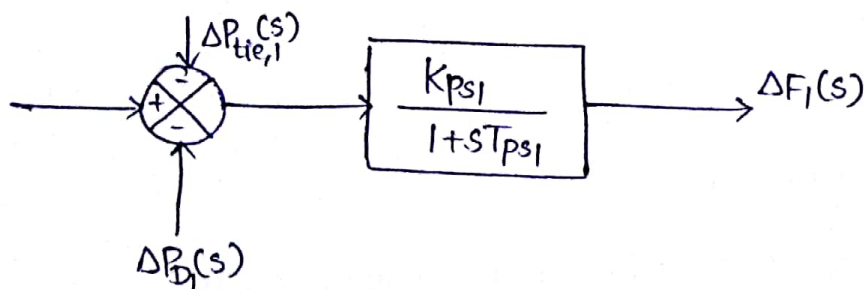
$$K_{ps1} = 1/B_1$$

$$T_{ps1} = 2H/B f_1'$$

$$\Delta F_1(s) = \left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{tie,1}(s) \right] \times \left( \frac{K_{ps}}{1 + s T_{ps}} \right)$$

→ (5)

Comparing above equation of isolated control area case, the only one change is the appearance of signal  $\Delta P_{tie,1}(s)$  and is represented as



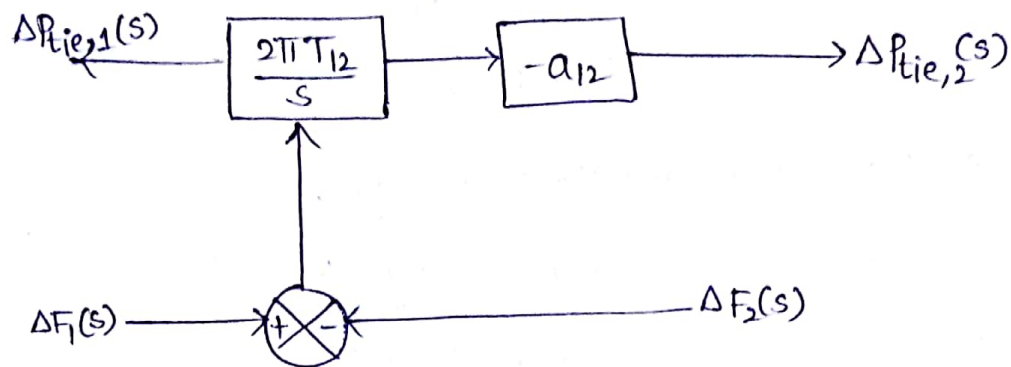
Taking laplace transform of eqn(2), we get

$$\Delta P_{tie,1}(s) = \left( \frac{2\pi T_{12}}{s} \right) [\Delta F_1(s) - \Delta F_2(s)]$$

Taking laplace transform of eqn(3), we get

$$\Delta P_{tie,2}(s) = -\frac{2\pi}{s} \alpha_{12} T_{12} [\Delta F_1(s) - \Delta F_2(s)]$$

The corresponding block diagram is shown in figure.



### Area control Error (ACE)

In case of an isolated area,

ACE is the change in area frequency which when used in integral control loop forced the steady state frequency error to zero.

In case of two-area control,

The steady state tie line power error can be made zero by introducing another integral control loop (one for each area).

This is accomplished by a single integrating block by redefining ACE as a linear combination of incremental frequency and tie-line power.

For control area 1

$$ACE_1 = \Delta P_{tie,1} + b_1 \Delta f_1$$

where the constant  $b_1$  is called "area frequency bias".

The above eqn can be expressed in laplace transform as

$$ACE_1(s) = \Delta P_{tie,1}(s) + b_1 \Delta f_1(s)$$

Similarly

For control area 2,

$$ACE_2(s) = \Delta P_{tie,2}(s) + b_2 \Delta f_2(s)$$

Fig. shows composite block diagram of two-area LFC (by combining basic block diagrams of two control areas)

$$\Delta P_{tie,1} + b_1 \Delta f_1 = 0 \quad (\text{input of integrating block } -\frac{K_{i1}}{s})$$

$$\Delta P_{tie,2} + b_2 \Delta f_2 = 0 \quad (\text{input of integrating block } -\frac{K_{i2}}{s})$$

$$\Delta f_1 - \Delta f_2 = 0 \quad (\text{input of integrating block } -\frac{2\pi T_{12}}{s})$$



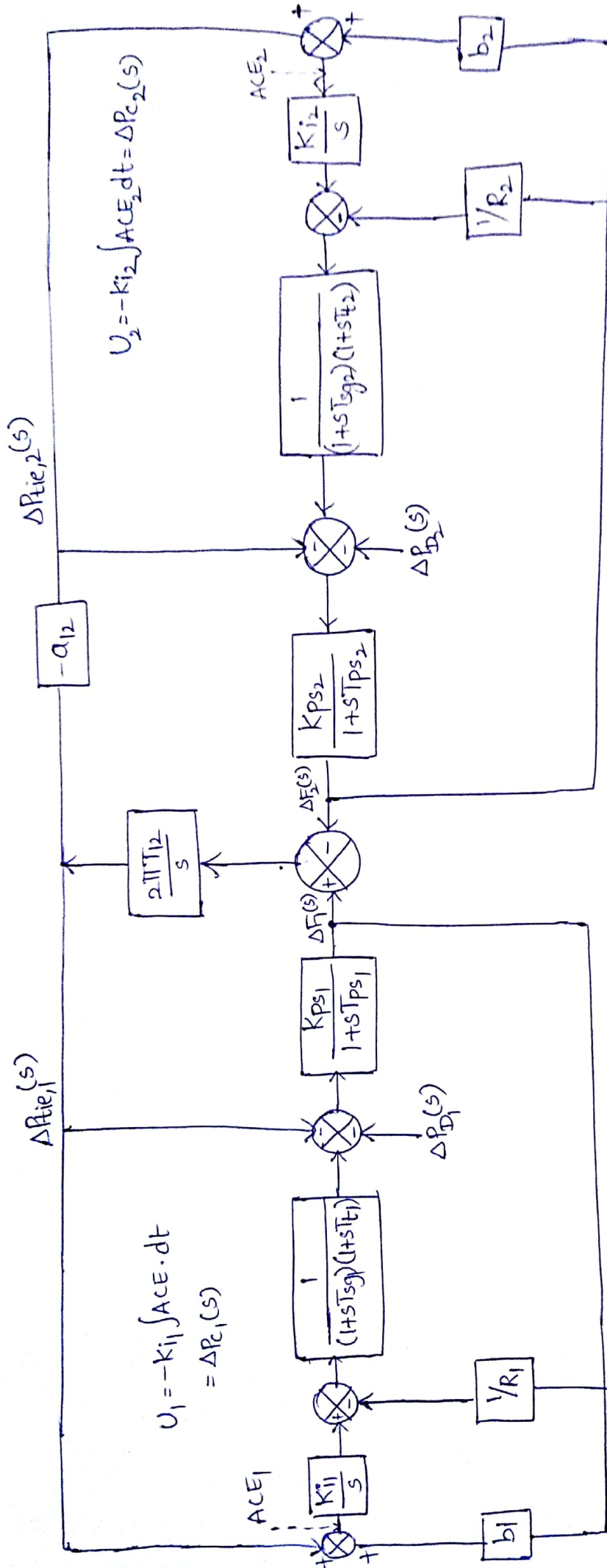


Fig. Composite block diagram of two-area load frequency control  
(Controlled case)  
(Feedback loops provided with integral of respective area control errors)

## Dynamic response of two-area system:

To describe the dynamic response of the two-area system, a system of seventh order differential equations is required. The solution of these equations would be tedious.

The analysis of two-area system is based on the following assumptions.

1. Consider the case of two (equal) identical areas.
2. The time constants of speed governor and turbine are  $T_{sg} = T_t = 0$  for both the areas.
3. The damping constants of two areas are neglected.  
i.e.,  $B_1 = B_2 = 0$

We know that,

Incremental power balance equation for area 1 & 2 is given by

$$\Delta P_{G1} - \Delta P_{D1} = \frac{2H_1}{f^0} \frac{d}{dt} (\Delta f_1) + B_1 \Delta f_1 + \Delta P_{tie,1} \longrightarrow \textcircled{1}$$

$$\Delta P_{G2} - \Delta P_{D2} = \frac{2H_2}{f^0} \frac{d}{dt} (\Delta f_2) + B_2 \Delta f_2 + \Delta P_{tie,2} \longrightarrow \textcircled{2}$$

With above assumptions, eqns  $\textcircled{1}$  &  $\textcircled{2}$  can be written as

$$\Delta P_{G1} - \Delta P_{D1} = \frac{2H_1}{f^0} \frac{d}{dt} (\Delta f_1) + \Delta P_{tie,1} \longrightarrow \textcircled{3}$$

$$\Delta P_{G2} - \Delta P_{D2} = \frac{2H_2}{f^0} \frac{d}{dt} (\Delta f_2) + \Delta P_{tie,2} \longrightarrow \textcircled{4}$$

Taking laplace transform of eqns  $\textcircled{3}$  &  $\textcircled{4}$  and re-arranging, we get

$$\Delta F_1(s) = \frac{f^0}{2H_1 s} [\Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{tie,1}(s)] \longrightarrow \textcircled{5}$$

$$\Delta F_2(s) = \frac{f^0}{2H_2 s} [\Delta P_{G2}(s) - \Delta P_{D2}(s) - \Delta P_{tie,2}(s)] \longrightarrow \textcircled{6}$$

From the block-diagram, we have

$$\Delta P_{G1}(s) = \frac{-1}{R} \Delta F_1(s) \longrightarrow (7)$$

$$\Delta P_{G2}(s) = \frac{-1}{R} \Delta F_2(s) \longrightarrow (8)$$

( $\because R_1 = R_2 = R$  for identical areas)

We know that

$$\Delta P_{tie,1}(s) = \frac{2\pi T_{12}}{s} [\Delta F_1(s) - \Delta F_2(s)] \longrightarrow (9)$$

$$\Delta P_{tie,2}(s) = \frac{-2\pi a_{12} T_{12}}{s} [\Delta F_1(s) - \Delta F_2(s)] \longrightarrow (10)$$

The above eqn can be written as

$$\Delta P_{tie,2}(s) = -a_{12} \cdot \Delta P_{tie,1}$$

$$\boxed{\Delta P_{tie,2} = -\Delta P_{tie,1}} \longrightarrow (11)$$

( $\because a_{12} = \frac{P_{r1}}{P_{r2}} = 1$ , since two areas are identical)

By solving eqns (5) to (11), we get

$$\Delta P_{tie,1}(s) = \frac{\Delta P_{D2}(s) - \Delta P_{D1}(s)}{\left[ s^2 + \left( \frac{f^0}{2RH} \right) s + \frac{2\pi f^0 T_{12}}{H} \right]} \cdot \frac{\pi f^0 T_{12}}{H}$$

From the above equation, the following observations can be made.

i) The denominator is of the form

$$s^2 + 2\alpha s + \omega^2$$

where  $\alpha = \frac{f^0}{4RH}$  ;  $\omega = \sqrt{\frac{2\pi f^0 T_{12}}{H}}$

The values of  $\alpha$  and  $\omega^2$  are real and positive.

Hence, it can be concluded that from the roots of the characteristic equation, the response is stable and damped

ii) If  $\alpha = \omega_n$ , system is critically damped.



ii) If  $\alpha > \omega_n$ , system becomes overdamped.

iii) If  $\alpha < \omega_n$ , system becomes underdamped.

The roots are  $s_{1,2} = -\alpha \pm j\sqrt{\omega_n^2 - \alpha^2}$

$$= -\alpha + j\omega_n \sqrt{1 - \left(\frac{\alpha}{\omega_n}\right)^2}$$

$$s_{1,2} = -\alpha \pm j\omega_0$$

where  $\alpha$  = damping factor (or) decrement of attenuation

$\omega_0$  = damped angular frequency

$$= \sqrt{\frac{2\pi f^2 T_{12}}{H} - \left(B + \frac{1}{R}\right)^2 \frac{f^2}{16H^2}}$$

State-variable model - Two area system:

Dynamic response is difficult to obtain by transfer function approach because of

→ Complexity of blocks

→ Multiple inputs ( $\Delta P_{D1}, \Delta P_{D2}$ )

→ Multiple outputs ( $\Delta P_{tie,1}, \Delta P_{tie,2}, \Delta f_1, \Delta f_2$ )

A more organized and more conveniently carried out analysis is through the state space approach.

Formulation of state-space model for two-area system is as follows.

The state variables are:

$$x_1 = \Delta f_1 ; \quad x_4 = \Delta f_2 ; \quad x_8 = \int ACE_1 dt$$

$$x_2 = \Delta P_{G1} ; \quad x_5 = \Delta P_{G2} ; \quad x_9 = \int ACE_2 dt$$

The control variables are denoted by  $u_1$  and  $u_2$

$$u_1 = \Delta P_{C1} ; u_2 = \Delta P_{C2}$$

The disturbance variables are denoted by  $w_1$  and  $w_2$

$$w_1 = \Delta P_{D1} ; w_2 = \Delta P_{D2}$$

→ For formulating the state variable model for this purpose the conventional feedback loops are opened and each time constant is represented by a separate blocks as shown in fig.

→ Before presenting the optimal design, we must formulate the state model. This is achieved below by writing the differential equations describing each individual blocks as shown in fig. in terms of state variables.

(Note: The differential equations are written by replacing  $s$  by  $\frac{d}{dt}$ )

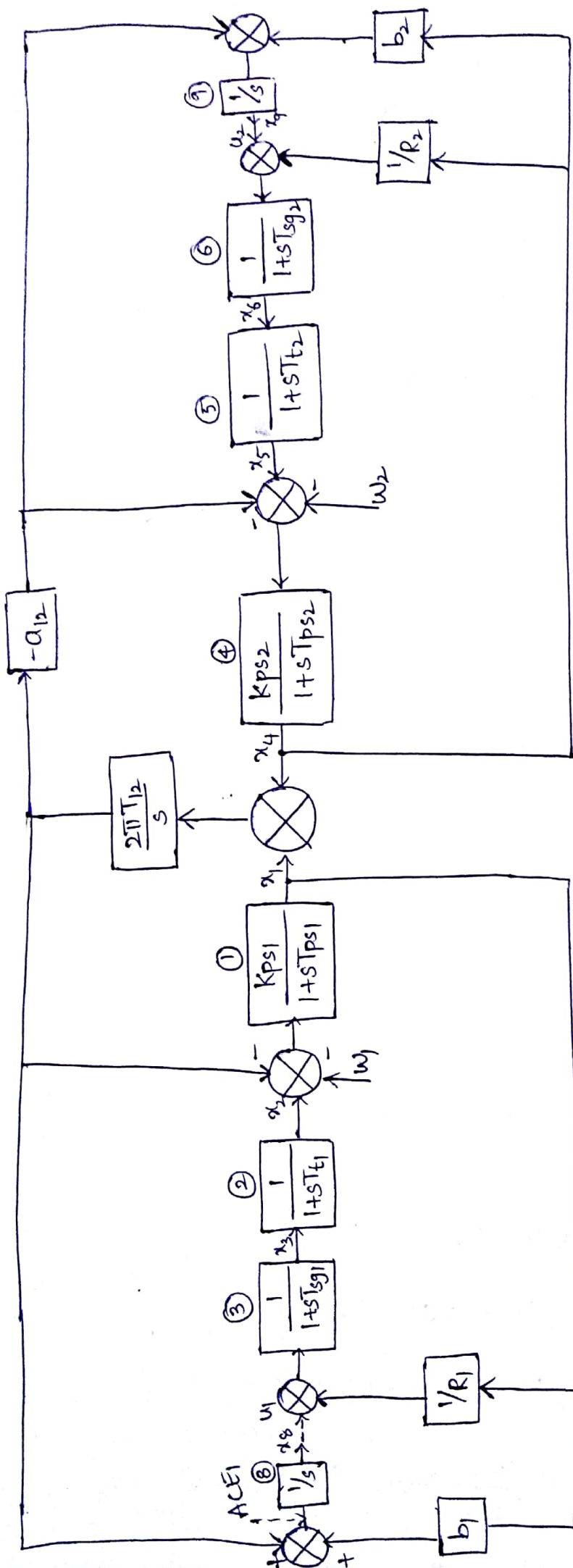


Fig. State space model of two-area system.



For block-1:

$$x_1 = \frac{K_{ps1}}{1+sT_{ps1}} (x_2 - x_7 - w_1)$$

$$x_1 + s x_1 T_{ps1} = K_{ps1} (x_2 - x_7 - w_1)$$

$$\dot{x}_1 = \frac{1}{T_{ps1}} [K_{ps1} (x_2 - x_7 - w_1) - x_1]$$

$$\dot{x}_1 = \frac{-1}{T_{ps1}} x_1 + \frac{K_{ps1}}{T_{ps1}} x_2 - \frac{K_{ps1}}{T_{ps1}} x_7 - w_1 \frac{K_{ps1}}{T_{ps1}} \longrightarrow \textcircled{1}$$

For block 2:

$$x_2 = \frac{1}{1+sT_{t1}} \cdot x_3$$

$$x_2 + s x_2 T_{t1} = x_3$$

$$\dot{x}_2 T_{t1} = x_3 - x_2$$

$$\dot{x}_2 = \frac{-1}{T_{t1}} x_2 + \frac{1}{T_{t1}} x_3 \longrightarrow \textcircled{2}$$

For block 3:

$$x_3 = \frac{1}{1+sT_{sg1}} (u_1 - \frac{1}{R_1} x_1)$$

$$x_3 + s x_3 T_{sg1} = u_1 - \frac{1}{R_1} x_1$$

$$\dot{x}_3 T_{sg1} = -\frac{1}{R_1} x_1 - x_3 + u_1$$

$$\dot{x}_3 = \frac{-1}{R_1} \cdot \frac{1}{T_{sg1}} \cdot x_1 - \frac{1}{T_{sg1}} x_3 + \frac{1}{T_{sg1}} u_1 \longrightarrow \textcircled{3}$$

For block 4:

$$x_4 = \frac{K_{ps2}}{1+sT_{ps2}} (x_5 + a_{12} x_7 - w_2)$$

$$x_4 + s x_4 T_{ps2} = [K_{ps2} (x_5 + a_{12} x_7 - w_2)]$$

$$\dot{x}_4 T_{ps2} = [K_{ps2} (x_5 + a_{12} x_7 - w_2)] - x_4$$

$$\dot{x}_4 = \frac{K_{ps2}}{T_{ps2}} \cdot x_5 + \frac{K_{ps2}}{T_{ps2}} a_{12} x_7 - \frac{K_{ps2}}{T_{ps2}} w_2 - \frac{1}{T_{ps2}} \cdot x_4$$

For block 5:

$$x_5 = \left( \frac{1}{1+sT_{t2}} \right) \cdot x_6$$

$$x_5 + s x_5 \cdot T_{t2} = x_6$$

$$\dot{x}_5 T_{t2} = -x_5 + x_6$$

$$\dot{x}_5 = \frac{-1}{T_{t2}} x_5 + \frac{1}{T_{t2}} x_6$$

For block 6:

$$x_6 = \left( \frac{1}{1+sT_{sg2}} \right) \left( u_2 - \frac{1}{R_2} x_4 \right)$$

$$x_6 + s x_6 T_{sg2} = \frac{-1}{R_2} x_4 + u_2$$

$$\dot{x}_6 T_{sg2} = \frac{-1}{R_2} x_4 - x_6 + u_2$$

$$\dot{x}_6 = \frac{-1}{T_{sg2} R_2} x_4 - \frac{1}{T_{sg2}} x_6 + \frac{1}{T_{sg2}} u_2$$

For block 7:

$$x_7 = \frac{2\pi T_{12}}{s} (x_1 - x_4)$$

$$s x_7 = 2\pi T_{12} (x_1 - x_4)$$

$$\dot{x}_7 = 2\pi T_{12} x_1 - 2\pi T_{12} x_4$$

For block 8:

$$x_8 = \frac{1}{s} (b_1 x_1 + x_7)$$

$$s x_8 = b_1 x_1 + x_7$$

$$\dot{x}_8 = b_1 x_1 + x_7$$

For block -9

$$x_9 = \frac{1}{s} (b_2 x_4 - a_{12} x_7)$$

$$s x_9 = b_2 x_4 - a_{12} x_7$$

$$\dot{x}_9 = b_2 x_4 - a_{12} x_7$$

The nine equations can be organized in following vector matrix form.

$$\dot{x} = Ax + Bu + Fw$$

where  $x = [x_1 \ x_2 \ \dots \ x_9]^T$  = state vector

$u = [u_1 \ u_2]^T$  = control vector

$w = [w_1 \ w_2]^T$  = disturbance vector

while the matrices A, B and F are defined below.

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix} & \begin{bmatrix} \frac{-1}{T_{ps1}} & \frac{K_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & \frac{-K_{ps1}}{T_{ps1}} & 0 & 0 \\ 0 & \frac{-1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{R_1 T_{sg1}} & 0 & \frac{-1}{T_{sg1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{ps2}} & \frac{K_{ps2}}{T_{ps2}} & 0 & \frac{a_{12} K_{ps2}}{T_{ps2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{R_2 T_{sg2}} & 0 & \frac{-1}{T_{sg2}} & 0 & 0 & 0 \\ 2\pi T_{12} & 0 & 0 & -2\pi T_{12} & 0 & 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & b_2 & 0 & 0 & 0 & -a_{12} & 0 \end{bmatrix} \end{matrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & \frac{1}{T_{sg1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{sg2}} & 0 & 0 & 0 \end{bmatrix}$$

$$F^T = \begin{bmatrix} \frac{-K_{ps1}}{T_{ps1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-K_{ps2}}{T_{ps2}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



## Load frequency controllers:

Proportional plus integral (control of single area and its block diagram representation, steady state response - Load frequency control and Economic dispatch control).

## Basic concept of control area

In the early days electric power systems were usually operated as individual units because they started as isolated systems. The demand for large blocks of power and increased reliability suggested the interconnection of neighbouring plants.

Interconnection is advantageous because fewer machines are required as a reserve for operation at peak loads and fewer machines running without load are required to take care of sudden, unexpected jumps in load (spinning reserve). Therefore, all generating plants are interconnected to form a regional grid, state grid and the national grid.

Load dispatch centres are required for the control of power flow in these grids.

It is feasible to divide a very large power system, say a national grid into sub-areas in which all the generators are assumed to be tightly coupled i.e., they swing in unison with change in load (or) due to speed changer settings.

### Definition of control area:

The area in which all the generators are tightly coupled together so as to form a coherent group.

i.e., all the generators respond in unison to changes in load (or) speed changer settings. Such area is called "control area".

→ In control area, the frequency is assumed to be same throughout in static as well as dynamic conditions.

→ For purpose of developing a suitable control ~~systems~~ strategy, control area can be reduced to a single speed-governor, turbo governor and load system.

### Area control Error (ACE)

The change in frequency in a given control area is known as "Area Control Error".

### Proportional plus integral load frequency control:

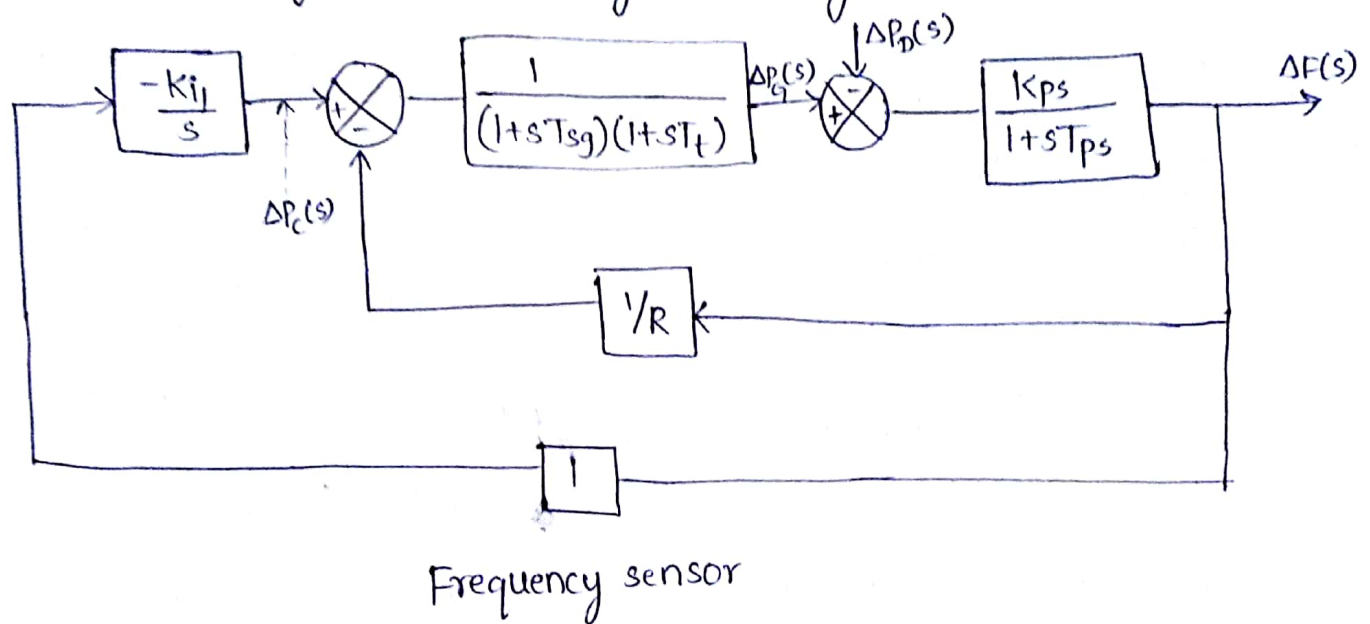
As so much change in frequency cannot be tolerated.

In fact, it is expected that steady change in frequency will be zero. by introducing integral controller. The steady state frequency can be brought back to prescribed value (or scheduled value) by adjusting speed changer settings as the load on system is changed.

→ The speed changer setting be adjusted automatically by monitoring the frequency changes.

For this purpose, a signal from  $\Delta f$  is fed through an integrator (i.e.,  $\frac{K_i}{s}$ ) to the speed changer.

The system now modifies to a proportional plus integral controller. The system block diagram configuration is shown



From the block diagram,

$$\Delta P_c(s) = -\frac{K_i}{s} \Delta F(s) \longrightarrow (1)$$

$$\left[ -\frac{1}{R} \Delta F(s) - \frac{K_i}{s} \Delta F(s) \right] \cdot \frac{1}{(1+sT_{sg})(1+sT_t)} = \Delta P_g(s)$$

$$-\left[ \frac{1}{R} + \frac{K_i}{s} \right] \cdot \frac{1}{(1+sT_{sg})(1+sT_t)} \cdot \Delta F(s) = \Delta P_g(s) \longrightarrow (2)$$

$$\Delta F(s) = \left( \frac{K_p s}{1+sT_{ps}} \right) (\Delta P_g(s) - \Delta P_d(s)) \longrightarrow (3)$$

$$\Delta F(s) = -\left( \frac{K_p s}{1+sT_{ps}} \right) \cdot \Delta P_d(s) + \left( \frac{K_p s}{1+sT_{ps}} \right) \Delta P_g(s)$$

$$= -\left( \frac{K_p s}{1+sT_{ps}} \right) \cdot \Delta P_d(s) + \left( \frac{K_p s}{1+sT_{ps}} \right) \cdot \left\{ -\left[ \frac{1}{R} + \frac{K_i}{s} \right] \frac{1}{(1+sT_{sg})(1+sT_t)} \Delta F(s) \right\}$$

$$\Delta F(s) \left\{ 1 + \left[ \left( \frac{K_p s}{1+sT_{ps}} \right) \left( \frac{1}{R} + \frac{K_i}{s} \right) \frac{1}{(1+sT_{sg})(1+sT_t)} \right] \right\} = -\frac{K_p s}{1+sT_{ps}} \cdot \Delta P_d(s)$$



$$\Delta F(s) = \frac{\left( \frac{-K_{ps}}{1+sT_{ps}} \right)}{1 + \left[ \left( \frac{K_{ps}}{1+sT_{ps}} \right) \left( \frac{1}{R} + \frac{K_i}{s} \right) \cdot \frac{1}{(1+sT_{sg})(1+sT_t)} \right]} \cdot \Delta P_D(s) \rightarrow (4)$$

For a step (or sudden) change of load demand,

$$\Delta P_D(s) = \frac{\Delta P_D}{s}$$

The eqn(3) can be written as

$$\Delta F(s) = \frac{\left( \frac{-K_{ps}}{1+sT_{ps}} \right)}{1 + \left[ \left( \frac{K_{ps}}{1+sT_{ps}} \right) \left( \frac{1}{R} + \frac{K_i}{s} \right) \cdot \frac{1}{(1+sT_{sg})(1+sT_t)} \right]} \cdot \frac{\Delta P_D}{s}$$

The steady state change (error) in frequency is given by

$$\Delta f \Big|_{\text{steady state}} = \lim_{s \rightarrow 0} s \cdot \Delta F(s)$$

$$= 0$$

### LOAD FREQUENCY CONTROL AND ECONOMIC DISPATCH CONTROL

The following are the basic requirements needed for the control strategy.

1. The control on the system frequency is obtained through a closed-loop.

The stability is always a problem with a closed loop control. Hence, the first requirement is stability of CLC.

2. The frequency deviation due to a step-load change should return to zero. The control that offers zero frequency deviation is called 'Isochronous control'.

3. The control ~~magnitude~~ strategy should keep the magnitude of transient frequency error (or deviation) to a minimum.

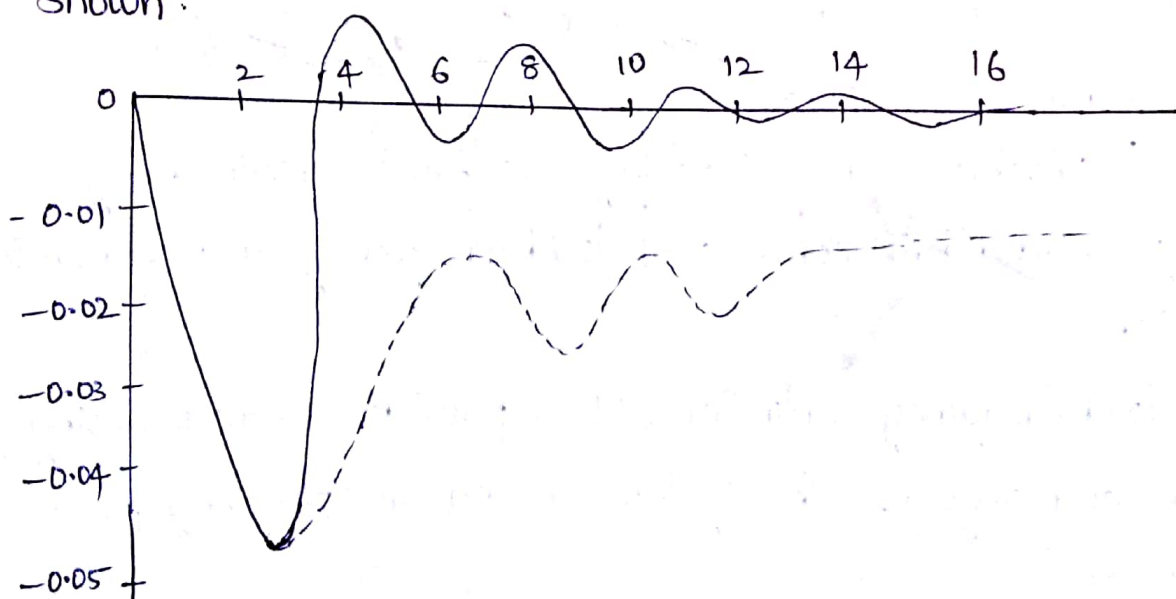
because no control can eliminate transient frequency error. The integral of frequency error should not exceed a certain maximum value.

4. The individual generators of the control area should divide (share) the load such that optimum economy is obtained.

The first three requirements can be met with aid of the integral control.

→ The order of load frequency controller of an isolated system is being of fourth order due to addition of integral loop.

The dynamic response of proportional plus integral controller can be obtained through digital computers as shown.



From the graph, it is concluded that the response of system is fast and zero steady state error can be achieved.

## Load frequency control and Economic dispatch control

Economic load dispatch and LFC play a vital role in modern power system.

In LFC, ...

zero steady-state frequency error, fast and dynamic response were achieved by integral controller action. But this control is independent of economic dispatch i.e., there is no control over the economic loadings of the individual units of the control area.

Some control over loading of individual units can be obtained by adjusting the gain factor ( $k_i$ ) of the integral signal of ACE as fed to individual units. But this is not satisfactory solution.

(In fact, some of the units in the process may even get overloaded).

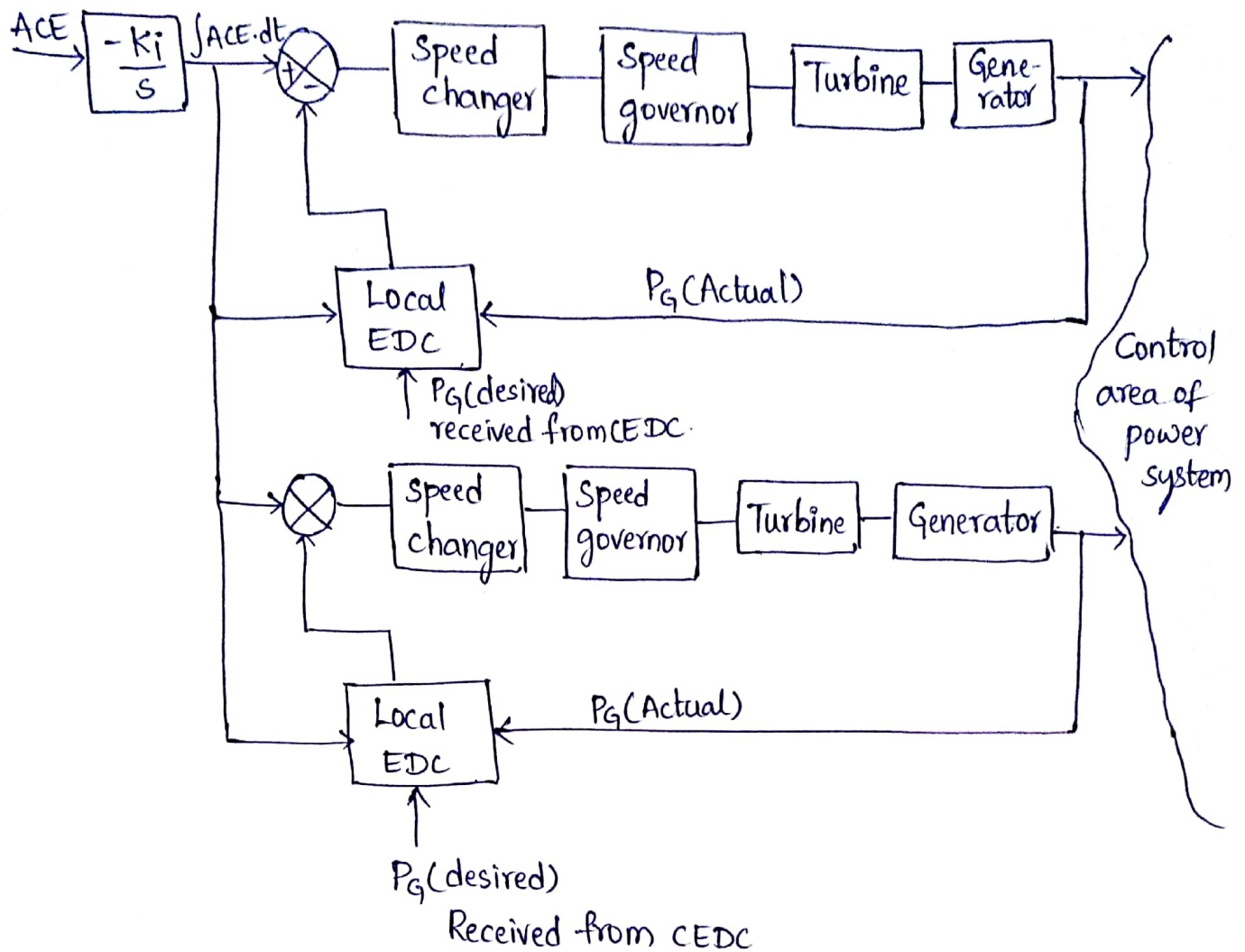
A suitable and satisfactory solution is obtained by using independent controls of load frequency and economic dispatch.

→ The load frequency controller (LFC) provides a fast-acting control and regulates the system around an operating point, where as

→ The economic load dispatch (EDC) provides a slow-acting control, which adjust speed-changer settings every minute in accordance with a command signal generated by central economic dispatch computer.



The schematic diagram shows both controls (LFC & EDC) for typical two units of a control area



CEDC - Central Economic Dispatch Controller

- The signal to change the speed changer setting is constructed in accordance with economic dispatch error (i.e.,  $P_G(\text{desired}) - P_G(\text{actual})$ ) suitably modified by signal representing integral ACE at that instant of time.
- The signal  $P_G(\text{desired})$  is computed by CEDC and is transmitted to the local EDC installed at each station.
- The system thus operates with economic dispatch error only for very short periods of time before it is readjusted.

# **UNIT – V**

## **REACTIVE POWER CONTROL**

### **5.1 Overview of Reactive power control:**

The economics of ac power transmission have always forced the planning engineer to transmit as much power as possible through a given transmission line. First, the dependence of load centers on the continuity of electrical supplies has become critical events. This means that the security (or) Reliability, of transmission circuits has needed to be continuously improved. Modern compensation methods have helped to make these improvements possible. Second, there has been extensive development of remote hydroelectric resources. The development of compensation schemes has helped to make ac transmission technically and economically competitive even in an age when the transmission alternative has made great strides also. The third planning constraint has been the difficulty of acquiring right – of – way for new transmission circuits. Increased pressure to maximize the utilization of both new and existing lines has helped to motivate the development and application of compensation systems.

The transmission of electrical power by ac is possible only if the following two fundamental requirements are satisfied.

- 1) Major synchronous machines must remain stably in synchronism.
- 2) Voltages must be kept near to their rated values.

Reactive power has been recognized as a significant factor in the design and operation of alternating current electric power systems for a long time. It has been observed that the impedances of the network components are predominantly reactive, the transmission of active power requires a difference in angular phase between the voltages at the sending and receiving points. The voltages at all the key points of the network. This control may be accomplished in large-part by the supply



(or) consumption of reactive power at these points. Reactive power have long been recognized they have recently acquired increased importance for at least two reasons, first the increasing pressures to utilize transmission capacity as much as possible and second the development of newer static types of control level reactive power compensator.

Thus a direct way of increasing power transfer capacity in transmission systems and of reducing voltage drop in distribution systems is to compensate part of the series inductive reactance by series capacitor. Operating problems have been encountered and applications to distribution systems have become rare, but the series capacitor remains the best way to increase transmission capacity in many cases. For purely sinusoidal single frequency voltages and currents, the concept is simple reactive current is the component out of phase with the voltage and there is a simple right triangle relation between active reactive and apparent power.

The need for adjustable reactive power compensation can be divided into three basic classes.

- 1) The need to maintain the stability of synchronous machines.
- 2) The need to control voltage within acceptable bounds about the desired steady state value to provide quality service to consumer loads.
- 3) The need to regulate voltage profiles in the network to prevent unnecessary flows of reactive power on transmission lines.

The static controlled reactive power sources almost always produce harmonics. The simple concept reactive power being the product of voltage and the out of phase component of current, where the reactive power by any direct measurement is practically zero, but the power factor is less than unity.

## **5.2 Reactive power compensation in transmission systems:**

Reactive power flow ( $Q$ ) is closely related with the voltage control. The apparent power  $S$ (KVA) is given by  $S = P \pm jQ$

Where  $S$  = Apparent power, KVA  
 $P$  = Real power, KW  
 $Q$  = Reactive power, KVAR.

The various equipments in the network 'absorb' (or) 'generate' reactive power.

They are 

- 1) Synchronous machine.
- 2) Shunt capacitors.

1) Synchronous machine:

An over excited synchronous motor working on no load, generates leading reactive power. It is a flexible reactive power source, which can generate variable amount of reactor power by changing the excitation.



**a) Transmission of reactive power:**

Over a long distance having additional losses for large size conductor and cost of the temperature.

Synchronous condenser:

Locate the synchronous condenser nearer to load constant, to reduce the losses in the system.

Industrial load lagging p.f provides static capacitors at the consumer's premises to supply the leading reactive power.

2) Shunt capacitors: A fixed capacitor is connected across the load at the consumers premises.

Volt amperes reactive are absorbed by inductive loads and  $Q$  for inductive loads is considered positive. Voltaamperes are supplied by capacitive loads and  $Q$  for capacitive load is considered negative.

In complex notations:

Complex power  $S$  is the product of voltage  $V$  and complex conjugate of  $I$

$$\text{i.e. } S = VI^* \text{ where } S = P + jQ.$$

Power  $P$  controls the active power which is converted into mechanical/heat (or) some other form.....and influence frequency  $f$ .

Reactive power  $Q$  is exchanged between inductive and capacitive loads in the network and influences the voltage in the network. Reactive power flow increases losses. Hence compensation is provided at each bus. The control of various bus voltage is achieved by supplying absorbing the reactive power requirements (KVAR) of respective busbars by means of series (or) shunt compensation. Compensation of reactive power means supplying/absorbing reactive vot-amperes.

Effect of reactive power flow on voltage at sending end and receiving end of transmission line:

$P$  = power transfer watts per phase

$Q$  = reactive power transfer VARs per phase

$V_s$  = Sending end voltage volts,

$V_R$  = receiving end voltage

$$\Delta V = V_s - V_R$$

= Drop in the line voltage

$R + jX$  = Series impedance of line/Ph

. The relation between  $V_s$ ,  $V_R$  and  $P$ ,  $Q$  is given by the equation.

$$\Delta V = V_S - V_R = \frac{RP + XQ}{V_R}$$

If the resistance R is neglected i.e.  $X \gg R$ , then

$$\Delta V = V_S - V_R = \frac{XQ}{V_R}$$

Hence voltage drop in line depends mainly on the flow of reactive power Q. The power angle  $\delta$  between  $V_R$  and  $V_S$  is proportional to

$$\delta \propto \frac{XP - RQ}{V_R} = \frac{XP}{V_R}$$

IF  $X \gg R$ , angle  $\delta$  depends mainly on P.

Thus,

Voltage is mainly controlled by reactive power flow power angle  $\delta$  is mainly controlled by real power flow. For voltage control, the flow of reactive power through the transmission line should be controlled. The flow of reactive power is controlled by injecting required VAR's into the system by means of

- Static shunt capacitors / reactors (SVS)
- Series capacitors
- Synchronous condensers (Compensators)

### 5.3 Advantages and disadvantages of different types of compensating equipment for transmission systems:

	Compensating equipment	Advantages	Disadvantages
1)	Switched shunt reactor	Simple in principle and construction	Fixed in value
2)	Switched shunt capacitor	Simple in principle and construction	Fixed in value switching transients.
3)	Series capacitor	Simple in principle performance relatively insensitive to location	Requires overvoltage protection and subharmonic filters, limited overload capability.
4)	Synchronous condenser	Has useful overload capability, fully controllable, low harmonics.	High maintenance requirement, slow control response, performance sensitive to location, requires strong foundations.
5)	Polyphase saturated reactor	Very rugged construction, large overload capability, No effect on fault level, low harmonics	Essentially fixed in value performance sensitive to location, Noisy.

6)	Thyristor – controlled reactor (TCR)	Fast response, Fully controllable, No effect on fault level can be rapidly repaired	Generates harmonics, performance sensitive to location
7)	Thyristor-switched capacitor (TSC)	Can be rapidly repaired after failures, No harmonics.	No inherent absorbing capability to limit overvoltages, complex buswork and controls, Low frequency resonances with system, Performance sensitive to location.

#### 5.4 Load compensation:

The compensation in power system is basically designed for absorption (or) generation of reactive power. There are two methods of compensation.

- 1) Load compensation
- 2) Line compensation

Compensator is connected at the terminals of the load is called load compensation. Compensator may be located at midpoint of the line (or) some suitable location uniformly distributed is called line compensation.

In an ideal ac power system, the voltage and frequency at every supply point would be constant and free harmonics, and the power factor would be unity. These parameters would be independent of the size and characteristics of consumer's loads. In these – phase systems, the degree to which the phase currents and voltage are balanced must also be included in the notion of quality of supply.

Load compensation is the management of reactive power to improve the quality of supply in an ac power systems. The term load compensation is used where the reactive power management is effected for single load, the compensating equipment usually being installed on the consumer's own premises near to the load. In load compensation there are three main objectives.

- 1) Power factor correction.
- 2) Improvement of voltage regulation
- 3) Load balancing

##### 1) Power factor connection:

The supply utilities also have good reasons for not transmitting unnecessary reactive power from generators to loads, their generators and distribution networks cannot be used at full efficiency, and the control of voltage in the supply can become more difficult.

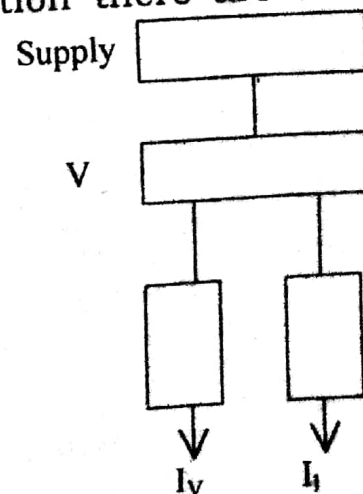


Fig.5.1



$$\begin{aligned}
 \text{Load current } I_1 &= V_{Y1} \\
 &= V(G_1 + jB_1) \\
 &= VG_1 + jVB_1 \\
 I_1 &= I_R + jI_X.
 \end{aligned}$$

It is an inductive load, where  $B_1$  is negative (-ve) and  $I_X$  is also negative, (-ve).

$$\begin{aligned}
 \text{Complex power} &= VI_1^* \\
 &= V[I_R - jI_X] \\
 &= VI_R - jVI_X
 \end{aligned}$$

$$\text{Complex} = P_R + jQ_1$$

Where,  $Q_1$  is positive (+ve) when  $I_X$  is negative for inductive load.

To provide compensation and improvement in power factor connect compensator across the load having a suitable reactive admittances such that over all power factor becomes unity.

## 2) Improvement of voltage regulation:

Voltage regulation becomes an important. All loads vary their demand for reactive power, although their differ widely in their range and rate of variation. The limits may vary from typically  $\pm 5\%$  averaged over a period of a few minutes (or) hours, to the much more constraints imposed where large, rapidly varying loads could produce voltage dips to the operation of protective equipment.

$$dv = E - V = Z_s I_L = (R_s + jX_s)I_L$$

$$VI_L = P_L + jQ_L$$

$$\Rightarrow I_L = \frac{P_L + jQ_L}{V}$$

$$\therefore dv \text{ (or) } \Delta V = (R_s + jX_s)I_L$$

$$= (R_s + jX_s) \left( \frac{P_L + jQ_L}{V} \right)$$

$$= \frac{P_L R_s + jP_L X_s + jQ_L R_s + j^2 Q_L X_s}{V}$$

$$= \frac{P_L R_s - Q_L X_s}{V} + j \left( \frac{P_L X_s + Q_L R_s}{V} \right)$$

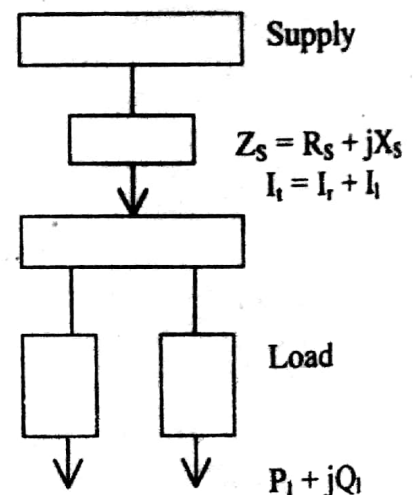


Fig.5.2.

$$\Delta V = \Delta V_R + j\Delta V_X.$$

$$\text{where } \Delta V_R = \frac{P_L R_s - Q_L X_s}{V}$$

$$\text{and } \Delta V_X = \frac{P_L X_s + Q_L R_s}{V}$$

By adding a compensating voltage for improving voltage regulation, a purely reactive compensator is designed. The voltage change  $\Delta V$  was a compensation voltage  $\Delta V_R$  inphase with voltage and  $\Delta V_X$  in perpendicular with voltage. The change in voltage 'V' depends on the real and reactive of the load i.e.  $P_L + jQ_L$ .

### 3. Load balancing:

ac power systems are three-phase, and are designed for balanced operation. Unbalanced operation gives rise to components of current in the wrong phase-sequence.

### 5.5 Specifications of load compensator:

The parameters and factors need to be considered when specifying a load compensator are summarized in the following.

- 1) Maximum continuous reactive power requirement, both absorbing and generating.
- 2) Over load rating and duration.
- 3) Rated voltage and limits of voltage between which the reactive power ratings must not be exceeded.
- 4) Frequency and its variation.
- 5) Voltage regulation required.
- 6) Special control requirements.
- 7) Response time of the compensator for a specified disturbance
- 8) Maintenance, spare parts, provision for future expansion.
- 9) Reliability of components
- 10) Performance with unbalanced supply voltages and / or with unbalanced load.

### 5.6 Uncompensated transmission lines:

A transmission line is characterized by four distributed circuit parameters:

- 1) Series resistance (r)
- 2) inductance (l)
- 3) Shunt conductance (g) and
- 4) capacitance (c)



All four parameters are functions of the line design, of the conductor size, type, spacing, height above ground, frequency and temperature. The characteristic behavior of the line is dominated by the series inductance and the shunt capacitance.

The fundamental equation governing the propagation of energy of a transmission line is the wave equation.

$$\frac{d^2 V}{dx^2} = \Gamma^2 V \dots \dots \dots (5.1)$$

$$\text{with } \Gamma^2 = (r + j\omega l) (g + j\omega c)$$

Frequency is assumed fixed, and  $V$  is the phasor voltage  $\hat{V} e^{j\omega t} / \sqrt{2}$  at any point on the line. If the line is assumed lossless, the general solution of equation (5.1)

$$V(x) = V_r \cos \beta(a - x) + jZ_0 I_r \sin \beta(a - x) \dots \dots \dots (5.2)$$

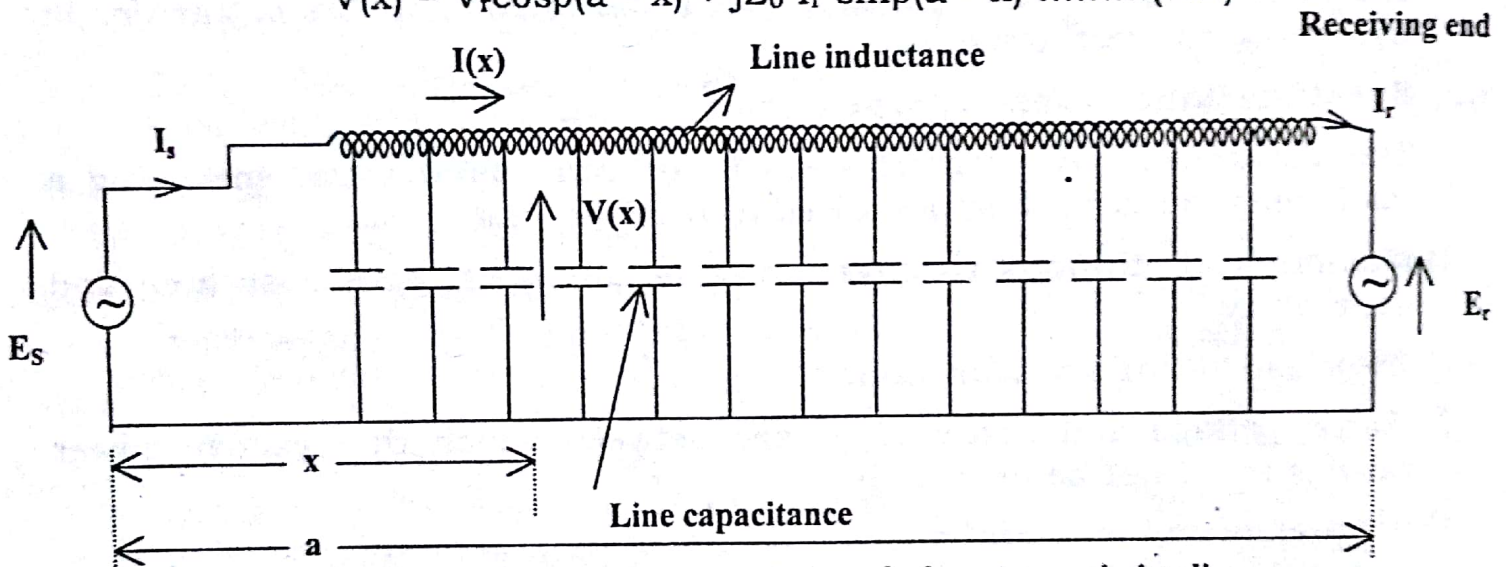


Fig.5.3. Lumped-element representation of a long transmission line

$$I(x) = j \left( \frac{V_r}{Z_0} \right) \sin \beta(a - x) + I_r \cos \beta(a - x) \dots \dots \dots (5.3)$$

Where  $\beta$  is derived from the propagation constant  $\Gamma$  by putting  $r = g = 0$

$$\Gamma = j\beta$$

$$\text{and } p = w \sqrt{lc} \dots \dots \dots (5.4)$$

$V$  and  $I$  are form standing waves because of the sinusoidal variation of both their real and imaginary parts of the line.

$$\text{Velocity of light} = u = 3 * 10^8 \text{ m/sec}$$

The constant  $Z_0$  in equation (5.2) is the surge impedance

$$Z_0 = \sqrt{\frac{l}{c}} \dots \dots \dots (5.4)$$



This impedance is also called the characteristic impedance.

For overhead lines, the positive - sequence value is  $400 \Omega$ .

The surge impedance is the apparent impedance of an infinitely long line, the ratio of voltage to current at any point along it.

$$\text{If } \frac{V_r}{I_r} = Z_0, \text{ then}$$

$$Z(x) = \frac{V(x)}{I(x)}$$

$$= \frac{V_r \cos \beta(a-x) + jZ_0 I_r \sin \beta(a-x)}{j \left( \frac{V_r}{Z_0} \right) \sin \beta(a-x) + I_r \cos \beta(a-x)}$$

$$= \frac{Z_0 I_r \cos \beta(a-x) + jZ_0 I_r \sin \beta(a-x)}{jI_r \sin \beta(a-x) + I_r \cos \beta(a-x)} = \frac{Z_0 I_r [\cos \beta(a-x) + j \sin \beta(a-x)]}{I_r [\cos \beta(a-x) + j \sin \beta(a-x)]}$$

$$= Z_0 \dots \dots \dots (5.5)$$

The line is said to have a flat voltage profile. While V and I are in phase with each other all along the line, both are rotated in phase.

The natural load (or) surge Impedance load SIL is

$$P_0 = \frac{V_o^2}{Z_0} \dots \dots \dots (5.6)$$

Where  $V_0$  is the nominal (or) rated voltage of the line. If  $V_0$  is the line-to-neutral voltage. Therefore, natural load of the uncompensated line increases with the square of the voltage. An advantage of operating the line at the natural load is that because of the flat voltage profile, the insulation is uniformly stressed at all points.

Voltage and current profiles: (Uncompensated line on open-circuit)

In equation (5.2) with  $I_r = 0$

$$\therefore V(x) = V_r \cos \beta(a-x) \dots \dots \dots (5.7)$$

and in equation (5.3) with  $I_r = 0$

$$\therefore I_x = j \left( \frac{V_r}{Z_0} \right) \sin \beta(a-x) \dots \dots \dots (5.8)$$

The voltage and current at the sending end are given by these equations with  $x = 0$

$$E_s = V_r \cos \theta \dots \dots \dots (5.9)$$

$$\begin{aligned}
 I_s &= j \left( \frac{V_r}{Z_0} \right) \sin \theta. & \text{Where } \theta &= \beta(a - x) \\
 &= j \left( \frac{E_s}{\cos \theta Z_0} \right) \sin \theta \\
 &= j \left( \frac{E_s}{Z_0} \right) \tan \theta \dots\dots\dots(5.10)
 \end{aligned}$$

$E_s$  and  $V_r$  are in phase, which is consistent with the fact that there is no power transfer.

$$\begin{aligned}
 \text{The voltage profile is } V(x) &= V_r \cos \beta(a - x) \\
 &= \frac{E_s}{\cos \theta} \cos \beta(a - x)
 \end{aligned}$$

$$V(x) = E_s \frac{\cos \beta(a - x)}{\cos \theta} \dots\dots(5.11)$$

Similarly the current profile is

$$I(x) = j \left( \frac{E_s}{Z_0} \right) \frac{\sin \beta(a - x)}{\cos \theta} \dots\dots\dots(5.12)$$

The terminal voltage are controlled to have the same magnitude,  $E_s = E_r$ ; with no power transfer the electrical conditions are the same at both ends. Therefore by symmetry  $I_s = -I_r$ .

The reactive power absorption capability of synchronous generators is limited for two reasons. First, underexcited operation increases the heating of the ends of the stator core. Second, the reduced field current reduces the internal emf of the generators, and this impairs stability.

In general, in the absence of compensating equipment, the synchronous machines must absorb (or) generate the difference between the reactive power of the line and the line and that of the local load.

## 5.7 Compensated transmission lines:

The theory of compensation basically related with the following

- 1) Supply systems
- 2) load
- 3) Compensater.

### 1) Supply systems:

Is modelled as an equivalent circuit with a voltage source in series with a series impedance.

### 2) Load:

Is considered as a constant, impedance (or) admittance.

**3) Compensator:** Is modelled as a variable impedance (or) admittance (or) a variable source of reactive power (or) reactive current

A compensation system ideally performs the following functions.

- 1) It improves stability by increasing the maximum transmissible power.
- 2) It helps produce a substantially flat voltage profile at all levels of power transmission.
- 3) It provides an economical means for meeting the reactive power requirements of the transmission system.

Compensated lines enable the transmission of the natural load over greater distances, and shorter compensated lines can carry loads greater than the natural load. A flat voltage profile can be achieved if the effective surge impedance of the line is modified so as to have a virtual value,  $Z_0'$ , for which the corresponding virtual natural load  $\frac{V_0^2}{Z_0'}$  is equal

to the actual load.

Compensations has divides into four types.

- 1) Passive compensation
- 2) Active compensation
- 3) Thyristor controlled capacitor
- 4) Thyristor controlled reactor

Passive compensators include shunt reactors, capacitors and series capacitors. These devices may be either permanently connected (or) switched. These compensators are used only for surge impedance compensation and line length compensation.

Active compensators are synchronous condensers only. These compensators are usually shunt-connected devices, which have the property of tending to maintain a substantially constant voltage at their terminals.

They are usually capable of continuous variation and rapid response.

In equipment development, activity is concentrated on the static reactive power controller (or) static compensator, to improve its efficiency, reliability, and response characteristics.

The surge impedance  $Z_0$  of an uncompensated is

$$Z_0 = \sqrt{\frac{l}{C}} = \sqrt{\frac{j\omega l}{j\omega c}}$$

A fundamental frequency this can be written as  $\sqrt{x_l x_c}$

$$\therefore Z_0 = \sqrt{\frac{l}{c}} = \sqrt{x_l x_c} \dots\dots\dots(5.13)$$



If a uniformly distributed shunt compensating inductance  $l_{vsh}$  is introduced, the effective value of the shunt capacitive admittance per mile becomes

$$(jwc)^1 = jwc + \frac{1}{j\omega l_{vsh}} \\ = jwc(1 - K_{sh}) \dots\dots\dots(5.14)$$

where  $k_{sh} \rightarrow$  is the degree of shunt compensation.

$$K_{sh} = \frac{1}{\omega^2 l_{vsh} c}$$

Substituting for  $(jwc)'$  in equation (5.13), the surge impedance has the effective (or) virtual value

$$Z_0' = \frac{Z_0}{\sqrt{1 - K_{sh}}} \dots\dots\dots(5.15)$$

Shunt inductive compensation, therefore, increases the virtual surge impedance, whereas shunt capacitive compensation reduces it. In a similar way the effect of uniformly distributed series capacitance

$$Z_0' = Z_0 \sqrt{1 - K_{se}} \dots\dots\dots(5.16)$$

Where  $K_{se}$  is the degree of series compensation

The parameters  $K_{sh}$  and  $K_{se}$  are a useful measure of the reactive power ratings required of the compensating equipment.

Combining the effects of shunt and series compensation,

$$Z_0' = Z_0 \sqrt{\frac{1 - K_{se}}{1 - K_{sh}}} \dots\dots\dots(5.17)$$

Corresponding to the virtual surge impedance  $Z_0'$  is a virtual natural load  $P_0'$ .

Given by  $\frac{V_0^2}{Z_0'}$ , so that

$$P_0' = P_0 \sqrt{\frac{1 - K_{sh}}{1 - K_{se}}} \dots\dots\dots(5.18)$$

The wave number  $\beta$  is also modified and has the virtual value

$$\beta' = \beta \sqrt{(1 - K_{sh})(1 - K_{se})} \dots\dots\dots(5.19)$$

The electrical length  $\theta$  is modified according to

$$\theta' = \theta \sqrt{(1 - K_{sh})(1 - K_{se})} \dots\dots\dots(5.20)$$

Where  $\theta = a\beta$  and  $\theta' = a\beta'$

## 5.8 Shunt compensation:

Shunt compensation with reactors increases the virtual surge impedance and reduces the virtual natural load. In practice, shunt compensating reactors cannot be uniformly distributed. They are connected at the ends of the line and at intermediate points – usually at intermediate switching substations. In the case of very long lines, at least some of the shunt reactors are permanently connected to the line in order to give maximum security against overvoltage in the event of a sudden rejection of load (or) open circuiting of the line.

If there is a sudden load-rejection (or) open-circuiting of the line, it may be necessary to disconnect them very quickly, to prevent them from increasing the voltage.

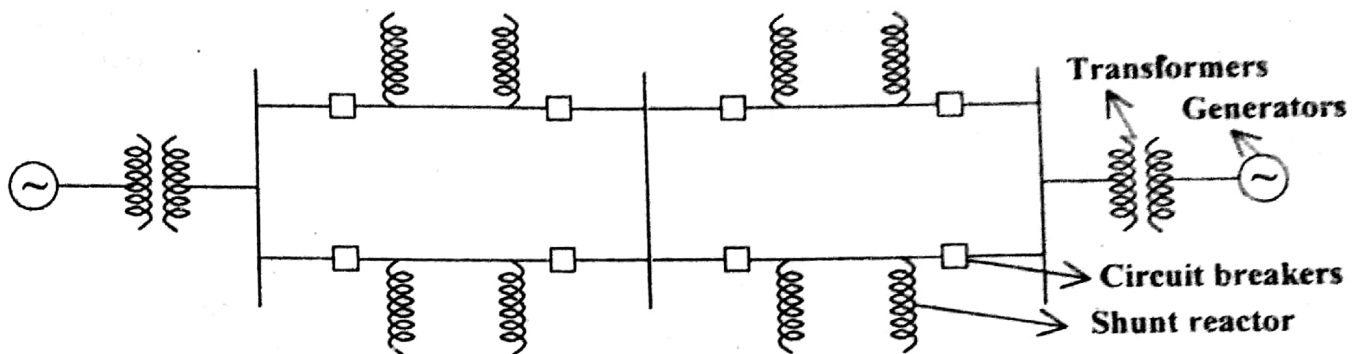


Figure 5.4. Arrangement of shunt reactors on a long distance high voltage ac transmission line

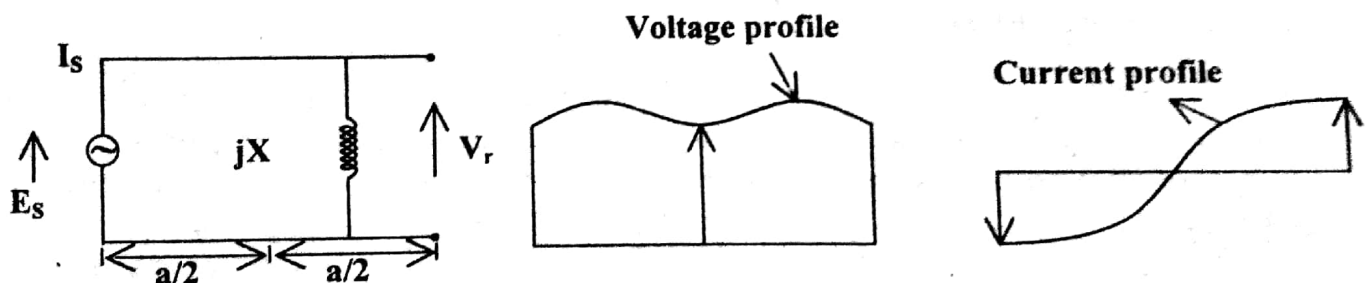


Figure 5.4. Voltage and current profiles of a shunt – compensated line at no load

Consider the simple circuit in figure 5.4 which has a single shunt reactor of reactance  $X$  at the receiving end and a pure voltage source  $E_s$  at the sending end. The receiving end voltage is given by

$$V_r = j X I_r \dots\dots(5.21)$$

From equation (5.2)  $E_s = V_r \cos \beta a + j Z_0 I_r \sin \beta a$

$$= V_r \left[ \cos \theta + \frac{Z_0}{X} \sin \theta \right] \dots\dots(5.22)$$

$E_s$  and  $V_r$  are, therefore, in phase, which is consistent with the fact that no real power is being transmitted. The receiving end voltage to be equal to the sending end voltage,  $X$  must be given by



$$X = Z_0 \frac{\sin \theta}{1 - \cos \theta} \dots\dots(5.23)$$

The sending end current is given by equation (5.3)

$$I_s = j \frac{E_s}{Z_0} \sin \theta + I_r \cos \theta \dots\dots(5.24)$$

Making use of equations (5.21) (5.22) and (5.23), this can be rearranged

$$\begin{aligned} I_s &= j \frac{E_s}{Z_0} \frac{1 - \cos \theta}{\sin \theta} \\ &= j \frac{E_s}{X} \\ &= - I_r \dots\dots(5.25) \end{aligned}$$

Since  $E_s = V_r$ . This means that the generator at the sending end behaves exactly like the shunt reactor at the receiving end in that both absorb the same amount of reactive power

$$\begin{aligned} Q_s - Q_r &= \frac{E_s^2}{X} \\ &= \frac{E_s^2}{Z_0} \left[ \frac{1 - \cos \theta}{\sin \theta} \right] \dots\dots(5.26) \end{aligned}$$

The charging current divides equally between the two halves of the line.

## 5.9 Series compensation:

Series compensation is to cancel part of the reactance of the line by means of series capacitors. This increases the maximum power, reduces the transmission angle at a given level of power transfer, and increases the virtual natural load. Reducing the transfer reactance between the ends of a line, the series capacitor finds two main classes of application.

- 1) It can be used to increase the power transfer on a line of any length. Sometimes a series capacitor is used to increase the load share on one of two (or) more parallel lines
- 2) It can be used to enable power to be transmitted stably over a greater distance than is possible without compensation.

The circuit would also be series resonant at the fundamental frequency, and it would be difficult to control transient voltages and currents during disturbances. The high voltage and reactive power absorption at the ends of the line can be corrected by means of shunt reactors.

The voltage on either side of the series capacitor is nearly 1.0 pu over practically the whole range of power transfer, implying that the shunt reactors could be permanently connected without disadvantage. The reactive absorption at the terminals is considerable reduced, and at high



values of  $p$  the terminal power factors become lagging, which may be transient stability because the internal generator voltages are increased.

## 5.10 Introduction to voltage stability problem in power systems:

Voltage stability is sometimes also called load stability. Power system stability may be defined as that property of the system which enables the synchronous machines of the system to respond to a disturbance from a normal operating condition so as to return to a condition where their operation is again normal. Stability studies are usually classified into three types depending upon the nature and order of magnitude of the disturbance.

- These are
- 1) Steady state stability
  - 2) Dynamic stability
  - 3) Transient state stability

The terms voltage instability and voltage collapse are often used interchangeable. The voltage instability is a dynamic process where in contrast to rotor angle stability, voltage dynamics mainly involves loads and the means for voltage control. Voltage instability limit is not directly correlated to the network maximum power transfer limit.

The concept of voltage stability is related to transient stability of a power system. The analysis of voltage stability normally requires simulation of the system modelled by non-linear differential algebraic equations. Comparisons of these types of stability is listed below.

	Steady state stability	Dynamic stability	Transient stability
1) Oscillations	No	Small	Large
2) Type of disturbance	Small, slow and gradual	Random load fluctuations	Large, sudden
3) Mathematical model	Non-linear algebraic equation	Linear differential equation	Non-linear differential equation
	$P_c \frac{ E_1   E_2 }{X} \sin \delta$	$\frac{M d^2 \delta}{dt^2} = P_a$	$\frac{M d^2 \delta}{dt^2} = P_l - P_m \sin \delta$

The concept of small disturbance voltage stability is related to steady state stability and can be analysed using small signal model of the system. Voltage security is the ability of a system, not only to operate stably, but also do remain stable following credible contingencies (or) load increases.

Certain situations in power system cause problems in reactive power flow which lead to system voltage collapse. Some of the situations that can occur are listed and explained below.

- 1) Long transmission lines
- 2) Radial transmission lines
- 3) Shortage of local reactive power

**1) Long transmission lines:**

In power systems, long lines with voltage uncontrolled buses at the receiving ends create major voltage problems during light load (or) heavy load conditions.

**2) Radial transmission lines:**

In a power system, most of the parallel EHV networks are composed of radial transmission lines. Any loss of an EHV line in the network causes an enhancement in system reactance. Under certain conditions the increases in reactive power delivered by the lines to the load for a given drop in voltage, is less than the increase in reactive power required by the load for the same voltage drop.

**3) Shortage of local reactive power:**

There may occur a disorganised combination of outage and maintenance schedule that may cause localised reactive power shortage leading to voltage control problems.

The voltage stability analysis for a given system state involves the following two aspects

- i) Proximity to voltage instability
  - ii) Mechanism of voltage instability
- i) Proximity of voltage instability: Instability may be measured in terms of physical quantities, such as load level, real power flow through a critical interface, and reactive power reserve. Possible contingencies such as a line outage, loss of a generating unit (or) a reactive power source must be given due consideration.
  - ii) Mechanism of voltage instability: The slower forms of voltage instability are normally analysed as steady state problems using power flow simulation as the primary study method. The two methods (i.e. pv curves and VQ curves) give steady state load ability limits which are related to voltage stability. Pv curves are useful for conceptual analysis of voltage stability and for study of radial systems.

### 5.11 Static compensators:

Static compensators are those which have no rotating parts and are used for surge impedance compensation and for compensation by sectioning a long transmission line. These are also used for load compensation where in these maintain constant voltage

- i) Under slowly varying load changes
- ii) Load rejection, outages of generator and line
- iii) Under rapidly varying loads.

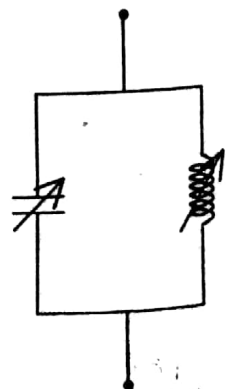


Figure 5.6



Fig. 5.6 shows an ideal static reactive power compensator. In actual practice, the reactive current is limited in both lagging and leading regions because of the current carrying capacity of the compensator. Some of the important compensators used in transmission and distribution networks are

- i) Thyristor controlled reactor (TCR)
- ii) Thyristor switched capacitors (TSC)
- iii) Saturated reactors.

**i) Thyristor controlled reactor (TCR):**

Figure 5.7 shows a basic thyristor controlled reactor. The controlled element is the reactor and the controlling element is the thyristor controller consisting of two oppositely poled thyristors which conduct every alternate half cycles of the supply frequency. The reactive power absorbed by the reactor will depend upon the instant of switching on the voltage wave.

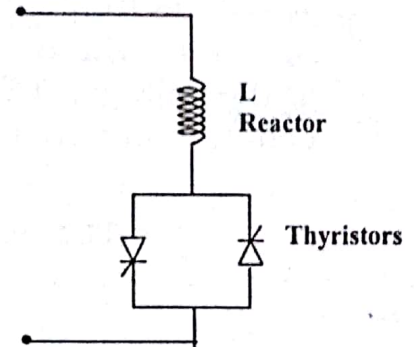


Fig. 5.7. Basic thyristor controlled reactor (TCR)

If the voltage is passing through its peak value at the instant of switching of thyristor, and as the delay angle increases between  $90^\circ$  to  $180^\circ$ . The fundamental component of current will be maximum when delay angle is  $90^\circ$  decreases with increase in delay angle between  $90^\circ$  and  $180^\circ$ .

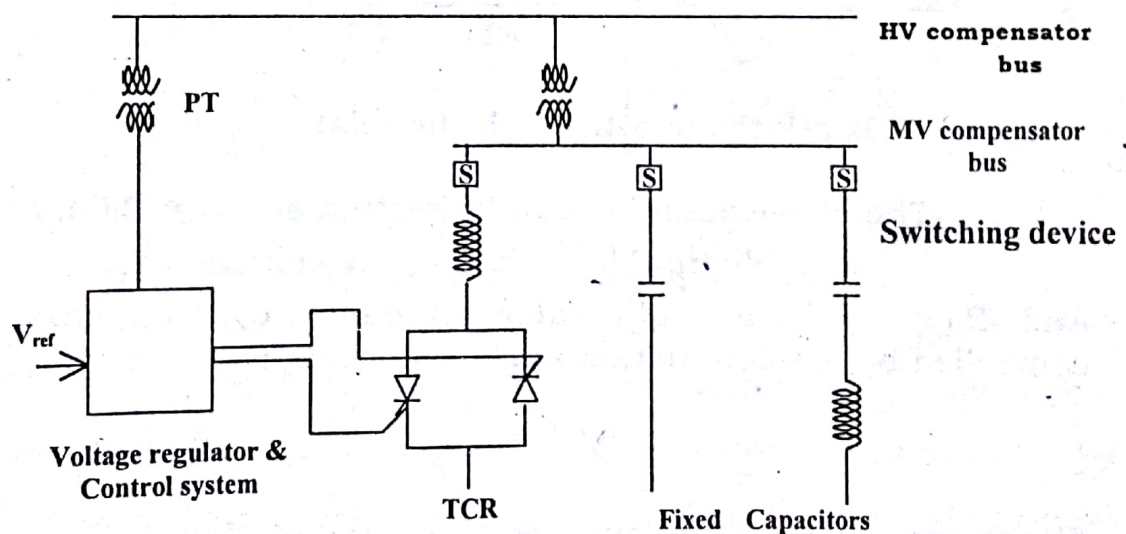


Figure 5.8 Thyristor controlled reactor (TCR)

To find the following features in the above Ckt.

- 1) A fixed shunt capacitor is parallel with the controlled susceptance. The fixed capacitors are usually tuned with small reactors to harmonic frequencies to absorb harmonics generated by the controlled susceptance (or) to avoid harmful resonances.
- 2) A step down transformer with significantly affects the performance of the transformer especially with respect to losses, harmonics and over voltages.



Increasing the delay angle (or) reducing the conduction angle decreases the power loss in both the reactors and the thyristors and generators harmonic currents. If both the thyristors are fired at the same angle, all odd order harmonics are produced.

## ii) Thyristor switched capacitors: (TSC)

Figure 5.10. shown the Thyristor switched capacitors (TSC) and figure 5.11 shows the current and voltage wave forms under transient free switching in and switching out of capacitor.

Assuming the time origin to coincide with a positive going zero crossing of the voltage, the delay angle ' $\alpha$ ' and the conduction angle ' $\rho$ ' so that the conduction period lies between  $\alpha$  and  $\alpha + \rho$ .

$$\text{i.e. } v = V_m \sin \omega t$$

Therefore, instantaneous current  $i$  is given as

$$i = \frac{1}{X_L} \int_{\alpha}^{\omega t} V_m \sin \omega t \, d(\omega t)$$

where  $X_L$  is the inductive reactance of the reactor at fundamental frequency

$$i = \frac{V_m}{X_L} [\cos \alpha - \cos \omega t] \text{ and } i = 0 \text{ for } \alpha + \rho < \omega t < \alpha + \pi$$

Using Fourier series, the fundamental component of current is given by

$$I_1 = \frac{\rho - \sin \rho}{\pi X_L} \cdot \frac{V_m}{\sqrt{2}}$$

$\rho$  is related to  $\alpha$  through the relation  $\alpha + \frac{\rho}{2} = \pi$

The above equation can be written as  $I_1 = B_1(\rho) \cdot V$

Where  $B_1(\rho) \rightarrow$  is the rms voltage.

And  $B_1(\rho) \rightarrow$  is an adjustable fundamental frequency susceptance controlled by the conduction angle according to the law

$$B_1(\rho) = \frac{\rho - \sin \rho}{\pi X_L}$$

The susceptance is maximum when  $\rho = \pi$  and its value is given as

$$B_1(\pi) = \frac{\pi - 0}{\pi X_L} = \frac{1}{X_L}$$

And its value is zero when  $\rho = 0$

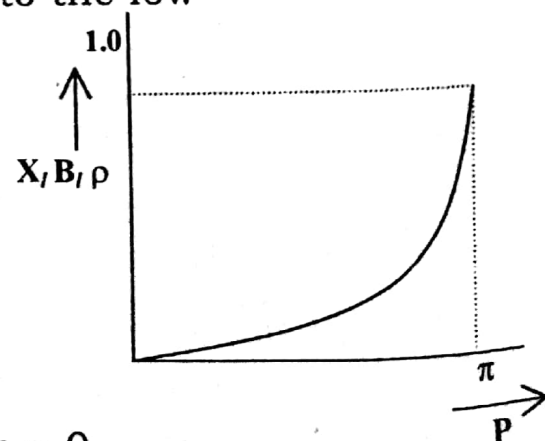


Fig. 5.9

The variation of  $B_1$  as a function of  $\rho$  is shown in figure 5.9.

Increasing the delay angle (or) reducing the conduction angle decreases the power loss in both the reactors and the thyristors and generators harmonic currents. If both the thyristors are fired at the same angle, all odd order harmonics are produced.

## ii) Thyristor switched capacitors (TSC)

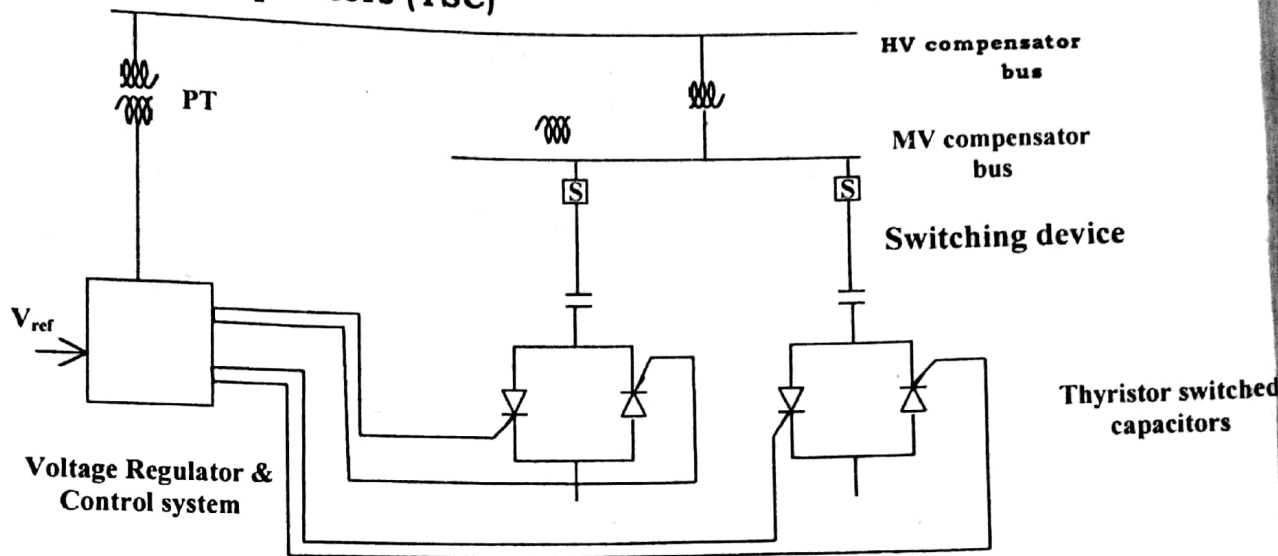


Figure 5.10 Thriristor switched capacitors (TSC)

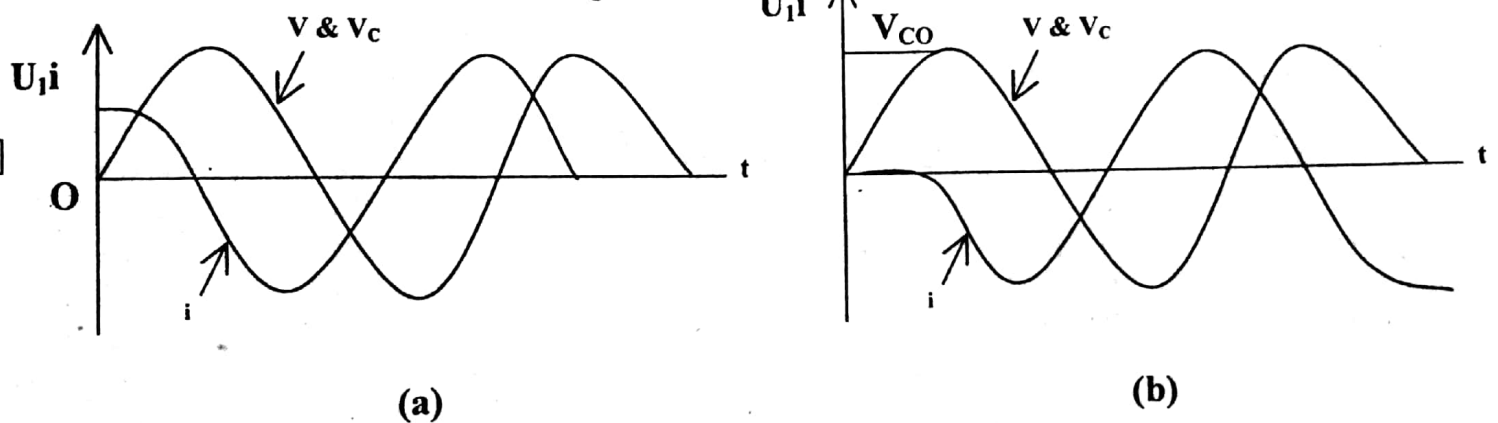


Fig. 5.11. Transient free switching operation  
(a) → Switching in (b) → Switching out

Fig 5.10 shown the Thyristor switched capacitors (TSC) and figure 5.11 shows the current and voltage wave forms under transient free switching in and switching out of capacitor.

Each capacitor always conducts for an integral number of cycles.

Normally the capacitors used are of the same capacity except one of the capacitors which has its susceptance half the susceptance of other capacitors. If there are  $(n - 1)$  number of equal capacitances and one of half value,  $2n$  number of combinations of capacitors is possible. In order to have transient free switching in (or) switching out of capacitor is charged to either the positive (or) the negative system peak voltage.

### iii) Saturated reactions:

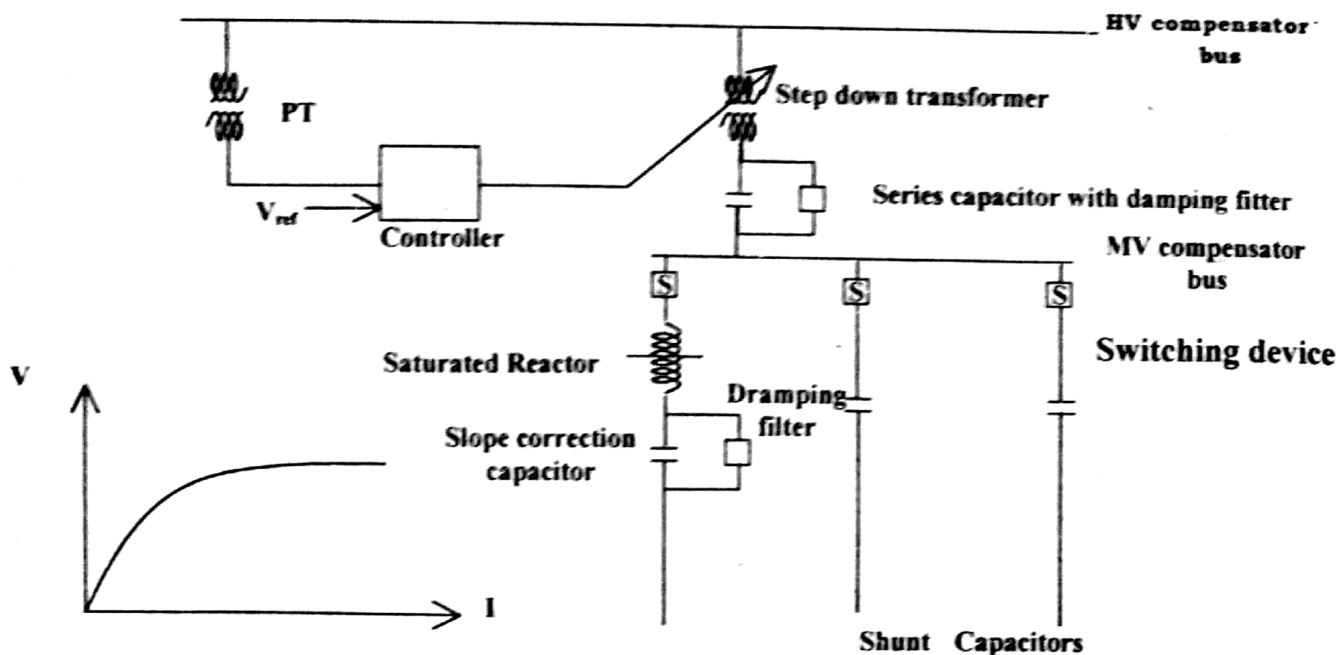


Fig. 5.13 Characteristic of a saturated reactor compensator

Figure 5.12 Saturated reactors

The plain saturated reactor is unsuitable for use in transmission systems as the voltage (or) the current contain lot of harmonics. Figure 5.13 shows a typical volt-amp characteristic of a saturated reactor, the slope of which depends upon the after-saturation inductance of the winding. The slope of the characteristic varies between 5 and 15%. The characteristic is linear above about 10% of the rated current. The presence of slope-correction capacitors in series with the saturated reactor has been recognised as a possible source of ferro-resonant sub-harmonic oscillations. Therefore, in the design of static reactive compensators this phenomenon is always suppressed by a sub-harmonic damping by-pass circuit incorporating an adequately dimensioned resistor and a power frequency tuned -rejector circuit as shown in figure.

### 5.12 Series capacitors:

Both series and shunt capacitor banks are useful tools in improving system efficiency and power transfer capacity. Shunt capacitors generate reactive power and help to reduce the amount of reactive power that flows through the network. For maximum effectiveness in reducing line losses and voltage control, they are typically installed near the load.

Series capacitors are applied to compensate for the inductive reactance of the transmission line. They may be installed remote from the load. Their benefits are

- 1) improved system steady state stability
- 2) improved system transient stability
- 3) reduced transmission losses.
- 4) adjustment of line loadings



The voltage drop on a transmission line is approximately.

Proportional to the inductive reactance  $X$ , the reduction of this reactance is an obviously powerful tool in improving voltage conditions. This is done by providing capacitors in series with the line and is known as series compensation.

In series compensation of the line  $K = \frac{X_c}{X} * 100$

Where  $K$  = is know as percentage compenation

$X_c$  = capacitive reactance of the series capacitors in series with the line

$X$  = reactance of the transmission line.

Figure 5.14 shows the short radial feeder with series capacitor.

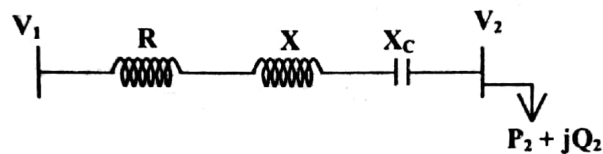


Fig. 5.14. Radial feeder with series capacitor

$$\text{Voltage drop (without series capacitor)} \Delta V = \frac{P_2 R + Q_2 X}{|V_2|}$$

$$\text{Voltage drop (with series capacitor)} \Delta V' = \frac{P_2 R + Q_2 (X - X_c)}{|V_2|}$$

$$\text{The capacitor raises the voltage} = \Delta V_c = \Delta V - \Delta V' = \frac{Q_2}{|V_2|} X_c$$

The capacitive reactance  $X_c$  used is generally less than the inductive reactance of the transmission line.

### 5.13 The reactive power balance and its effect on system voltage:

There is a strong inter-relation in the power system between reactive power flow and the system voltage. Let us consider the two bus system shown in figure 5.15. Load  $P_2 + jQ_2$  is connected to bus 2 and power is transferred from bus 1 through a short transmission line having resistance  $R$  and reactance  $X$ . Let us consider that the sending end voltage  $V_1$  phase to neutral is kept constant by the field control of the generator  $G_1$  and | or top changing transformer  $T$ .

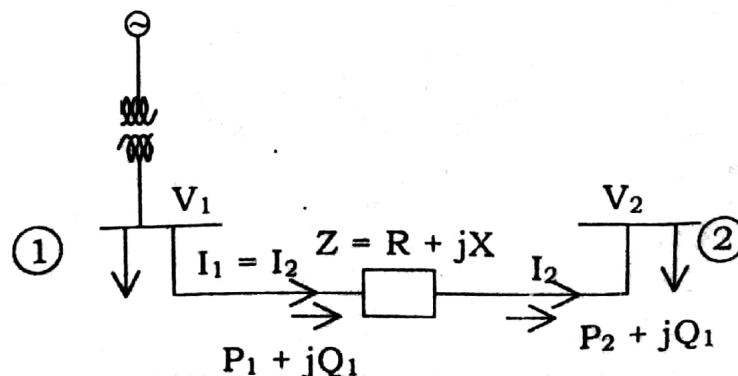


Fig. 5.15. Power flow from bus 1 to bus 2

The transmission line  $R \leq X$ , neglect the active power loss  $\Delta P_L$  simplifying the problem neglect the reactive power loss.  $\Delta Q_L$  of the line also.

$$\begin{aligned}\text{Above assumption, } S_2 &= P_2 + jQ_2 \\ &= P_1 + jQ_1\end{aligned}$$

$$\text{and the } V_2 = V_1 - I_1 (jX)$$

$$V_1 I_1^* = P_2 + jQ_2$$

$$(\text{or}) V_1^* I_1 = P_2 - jQ_2$$

$$\Rightarrow I_1 = \frac{P_2 - jQ_2}{V_1^*} = \frac{P_2 - jQ_2}{V_1}$$

( $V_1^* = V_1$  being the reference vector)

$$\therefore V_2 = V - j \frac{P_2 - jQ_2}{V_1} X$$

$$= \left( V_1 - \frac{Q_2 X}{V_1} \right) - j \frac{P_2}{V_1} X$$

$$= (V_1 - \Delta V) - j\delta V$$

$$\text{where } \Delta V = \frac{Q_2}{V_1} X$$

$$\text{and } \delta V = \frac{P_2}{V_1} X$$

When the reactive component of the load is decreased by connecting a static capacitor across the load and under this condition the new reactive load is

$$Q_2^1 = (Q_2 - Q_c)$$

Where  $Q_c$  is the reactive power of the static capacitor. A small change in the magnitude of the bus voltage, the real power at the bus does not change appreciably and similarly a small change in phase angle of the bus voltage, the reactive power does not change appreciably.

#### 5.14 Combined use of tap changing transformers and reactive power injection:

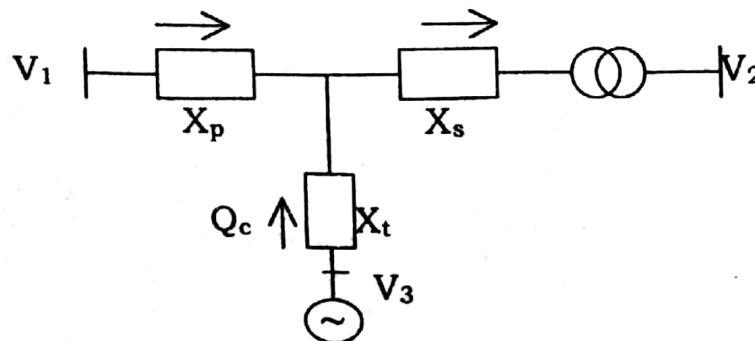


Fig 5.16. Equivalent circuit of combined use of tap changing transformers and reactive power injection

Equivalent of the transformer in terms of equivalent primary, secondary and tertiary reactances is shown in figure 5.16 along with off-nominal setting  $t$  at the receiving end.  $V_1$  and  $V_2$  are the phase voltages. Normally, tap setting are provided in steps for the range of  $\pm 20\%$

$$\text{Voltage drop } \Delta V = \frac{P_2 R + (Q_2 - Q_c) X_p}{|V_n|}$$

Where  $V_n \rightarrow$  is the phase voltage at the star point of the equivalent circuit of the three winding transformer.

$$= \frac{(Q_2 - Q_c) X_p}{|V_n|} \dots\dots\dots (5.27)$$

Resistance  $R$  of the transmission line and that of three-winding transformer is neglected.

$$\text{Similarly, quadrature voltage drop } \delta V = \frac{P_2 X_p}{|V_n|} \dots\dots\dots (5.28)$$

Using equations (5.27) and (5.28)

$$|V_1|^2 = (|V_n| + \Delta V)^2 + |\delta V|^2$$

$$= \left[ |V_n| + \frac{(Q_2 - Q_c) X_p}{|V_n|} \right]^2 + \left[ \frac{P_2 X_p}{|V_n|} \right]^2$$

$$\therefore |V_1|^2 = \left[ \frac{|V_n|^2 + (Q_2 - Q_c) X_p}{|V_n|} \right]^2 + \left[ \frac{P_2 X_p}{|V_n|} \right]^2$$

$$|V_1|^2 = \frac{1}{|V_n|^2} [|V_n|^2 + (Q_2 - Q_c) X_p]^2 + (P_2 X_p)^2$$

$$= \frac{1}{|V_n|^2} [|V_n|^4 + ((Q_2 - Q_c) X_p)^2 + 2 |V_n|^2 + (Q_2 - Q_c) X_p] + (P_2 X_p)^2$$

Solving  $|V_n|$

$$|V_n|^2 = \left[ \frac{|V_1|^2}{2} - (Q_2 - Q_c) X_p \right] \pm \sqrt{\frac{|V_1|^4}{4} - X_p |V_1|^2 (Q_2 - Q_c) - X_p^2 P_2^2} \dots\dots$$

### 5.15 Shunt reactors connected in high voltage transmission system:

Shunt capacitance in lines, and particularly in cables, actually have a surplus of reactive power during light load(or) no load conditions. During light load and under no load conditions, to connect Q-consuming elements.

Generally the shunt reactors are used in long EHV transmission line and the practical and economic reasons lead to concentrate these compensating elements at a few points along the line. If the linear shunt



reactors are switched out under heavy load conditions, the maximum power transfer capability is considerably increased, but voltage changes due to sudden load rejection are likely to be unacceptably high.

### 5.16 Line compensations:

Increases the steady state stability limit of the power system network. For increasing steady state stability limits, modify the reactance of the transmission network. Series compensation (or) a series capacitor to reduce the effective reactance of the transmission system.

- 1) **Line length compensation:** For any transmission line, three critical loadings
  - a) Natural impedance loading
  - b) Thermal loading
  - c) Steady state stability limit.For short lines, it is observed that the steady state stability limit will be greater than thermal loading limit.
- 2) **Medium lines compensation:** For un-compensated lines the voltage level is the main consideration for limiting the amount of power transfer.
- 3) **Long lines compensation:** Steady state stability limits the amount of power transfer. In the case of long lines it is essential that should try to employ suitable compensation to increase the steady state stability link.
- 4) **Ferranti effect:** In the case of long lines, under no load & light load conditions, the receiving end voltage is higher than sending end voltage. Variation in voltage levels for all operating conditions should be within permissible limit.
- 5) **Flat voltage profile:** Under excited operation of synchronous machines is to be avoided. Reactive power management is to be using suitable compensates.

### 5.17. Problems:

- 1) A load of  $(66 + j60)$  MVA at the receiving end is being transmitted via a single circuit 220 KV line, having resistance of 21 ohms and reactance of 34 ohms. The sending end voltage is maintained at 220 KV. The operating conditions of power consumers require that at this load voltage drop across the line should not exceed 5 percent. In order to reduce voltage drop, standard 1  $\phi$ , 0.66 KV, 40 KVAR capacitors are to be switched in series in each phase of the line. Determine the required number of capacitors, rated voltage and installed capacitors of the capacitor link. The losses in the line are neglected.

#### Solution:

$$\text{Resistance (R)} = 21 \, \Omega$$

$$\text{Reactance (X)} = 34 \, \Omega$$

$$P_2 = 22 \, \text{MW}$$

$$Q_2 = 20 \text{ MVAR}$$

$$|V_1| = \frac{220}{\sqrt{3}} = 127 \text{ KV}$$

$$\text{Voltage drop (without series capacitor)} = \Delta V = \frac{P_2 R + Q_2 X}{|V_2|}$$

$$\begin{aligned} |V_1| &= |V_2| \\ \therefore \Delta V &= \frac{(22 \times 21) + (20 \times 34)}{127} = 8.99 \text{ KV} \end{aligned}$$

$$\begin{aligned} \text{Permissible voltage drop } \Delta V_{\text{perm}} &= 5\% \times 127 \\ &= \frac{5}{100} \times 127 = 6.35 \text{ KV} \end{aligned}$$

$$\text{Voltage drop (with series capacitor)} = \therefore \Delta V' = \frac{P_2 R + (X - X_c) Q_2}{|V_2|}$$

But voltage drop with series capacitor should be equal to the permissible voltage drop.

$$\therefore \Delta V_{\text{perm}} = \Delta V'$$

$$\therefore \Delta V_{\text{perm}} = \frac{P_2 R + (X - X_c) Q_2}{|V_2|}$$

$$6.35 = \frac{(22 \times 21) + 20(34 - X_c)}{127}$$

$$\Rightarrow X_c = 16.65 \Omega$$

given rated load, the current in the line is

$$|I| = \frac{\sqrt{(66)^2 + (60)^2}}{\sqrt{3} \times 220} = 235.5 \text{ A}$$

$$\text{The rate current of the given capacitors } I_c = \frac{40}{0.66} = 60.6 \text{ A}$$

Therefore, the number of capacitors connected in parallel in one-phase must

$$\text{be greater than } m = \frac{235.5}{6.6} = 3.88 \approx 4$$

$$\therefore \text{The number of parallel connected capacitors} = 4$$

$$\text{The reactance of the given capacitor } X_{c1} = \frac{660}{60.6} = 10.9 \Omega$$

The reactance of each capacitor and the number of their parallel circuits m, determine the number of series connected capacitors n using the equation

$$\frac{X_{cl}}{m} * n = X_c$$

$$\Rightarrow \frac{10.9}{4} * n = 16.65$$

$$\Rightarrow n = 6.1 \approx 7$$

∴ The total number of capacitors in all the three phases are  
 $= 3 * 4 * 7 = 84$

The installed capacity of the capacitor bank is  $Q_{cb} = 84 * 40 = 3360 \text{ KVAR}$

The rated voltage of the capacitors bank is  $V_{cb} = 0.66 * n$   
 $= 0.66 * 7$   
 $= 4.62 \text{ KV}$

The rated current of the capacitors bank is  $I_{cb} = I_c * m$   
 $= 6.06 * 4$   
 $= 242.4 \text{ AV}$

Selected number of capacitors, the actual reactance of the bank is]

$$X_{cb} = \frac{X_{cl}}{m} * n$$

$$= \frac{10.9}{4} * 7 = 19.075 \Omega$$

The voltage drop in the line is  $\Delta V = \frac{(22 * 21) + 20(34 - 19.075)}{127} = 6 \text{ KV}$

2. With a 100 MVA generator operating at 85% power factor logging
- How many MVar will be produced?
  - What mega watt load should the machine be limited so that its MVA rating will not be exceeded?

**Solution:**

MVA rating of generator =  $100 * 0.85 = 85 \text{ MW}$

$$\phi = \cos^{-1}(0.85) = 31.788$$

(i)  $\text{MVar} = 85 * \tan(31.788) = 52.87 \text{ MVar}$

(ii) MW rating of generator = 85 MW

3. A single circuit three phase 220 KV, line operates on no load, voltage at the receiving end of the line is 205 KV. Find the sending end voltage, if the line has resistance of 21.7 ohms, reactance of 85.2 ohms and the total susceptance as  $5.32 * 10^{-4} \text{ mho}$ . The transmission line is to be represented by  $\pi$  method.



**Solution:**

$$R = 21.7 \Omega$$

$$X = 85.2 \Omega$$

$$\text{Susceptance} = 5.32 \times 10^{-4} \text{ mho}$$

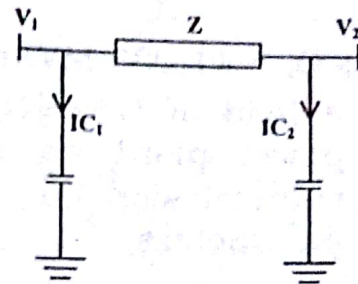
$$|V_1| = |V_2| - \frac{Q_c}{|V_2|} X$$

$$|V_2| = \frac{205}{\sqrt{3}} = 118.35 \text{ KV}$$

$$Q_c = |V_2|^2 \frac{W_c}{2} = (118.35)^2 \times \frac{5.32}{2} \text{ mho} = 3.716 \text{ MVar}$$

$$\therefore |V_1| = 118.35 - \frac{3.716}{118.35} \times 85.2$$

$$= 118.35 - 2.69 = 115.67 \text{ KV}$$



$$\text{Sending end voltage, line - to - line} = \sqrt{3} \times 115.67 = 200.34 \text{ KV}$$

4. The load at the receiving end of a three - phase, over head line is 25 MW, power factor 0.8 lagging, at a line voltage of 33 KV. A synchronous compensator is situated at the receiving end and the voltage at both ends of the line is maintained at 33 KV. Calculate the MVar of the compensator. The line has resistance 5 ohm per phase and inductive reactance 20 ohm per phase.

**Solution:**

The synchronous compensator of  $Q_c$  MVar is in parallel with the load, then

$$|V_1| = \sqrt{\left[ |V_2| + \frac{P_2 R + (Q_2 - Q_c) X}{|V_2|} \right]^2 + \left[ \frac{P_2 X - (Q_2 - Q_c) R}{|V_2|} \right]^2}$$

$$\text{Total load} = 25 \text{ MW}$$

$$P_2 = \frac{25}{3} = 8.33 \text{ MW / Phase}$$

$$Q_2 = 8.33 \times \tan^{-1}(\phi)$$

$$\text{Where } \phi = \cos^{-1}(0.8)$$

$$Q_2 = 8.33 \times \frac{3}{4} = 6.25 \text{ MVar / Phase}$$

$$|V_1| = |V_2| = \frac{33}{\sqrt{3}} = 19 \text{ KV}$$

$$R = 5 \Omega \text{ / Phase}$$

$$X = 20 \Omega \text{ / Phase}$$

Substituting the above given values, then the above expression is

$$19 = \sqrt{\left[19 + \frac{8.33 \cdot 5 + (6.25 - Q_c)20}{19}\right]^2 + \left[\frac{8.33 \cdot 20 - (6.25 - Q_c)5}{19}\right]^2}$$

$$\Rightarrow Q_c = 11.033 \text{ MVar}; \text{ Total reactive power} = 3 \cdot 11.0333 \cdot 33.099 \text{ MVar}$$

5. A load of  $(15 + j10)$  MVA is supplied with power from the busbars of power plant via a three phase 110 KV line, 100 KM lagging. The transmission line is represented by  $\pi$  - method and has the following parameters

$$R = 26.4 \Omega$$

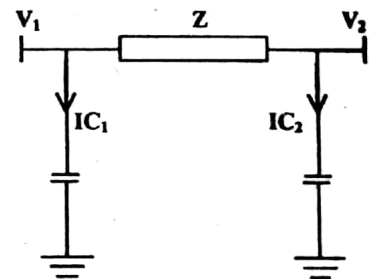
$$X = 33.9 \Omega$$

$$B = 219 \cdot 10^{-6} \Omega$$

Voltage across the power plant has bars  $V_1 = 116 \text{ KV}$ . Find the power consumed from the power plant bus bars.

**Solution:**

$$\begin{aligned} S_2 &= P_2 + jQ_2 \\ &= \frac{1}{3}(15 + j10) \\ &= 5 + j3.33 \text{ MVA} \end{aligned}$$



$$\text{Rated voltage per phase } |V_r| = \frac{110}{\sqrt{3}} = 63.5 \text{ KV}$$

$$\begin{aligned} Q_c &= |V_r|^2 \frac{WC}{2} \\ &= \frac{(63.5)^2 \cdot 219 \cdot 10^{-6}}{2} = 0.441 \text{ MVar} \end{aligned}$$

$$\begin{aligned} \text{Power at the receiving end of the line } S &= S_2 - Q_c \\ &= 5 + j3.33 - j0.441 = (5 + j2.89) \text{ MVA} \end{aligned}$$

$$\text{Power loss in the line } \Delta P_L = \frac{|S|^2}{|V_r|^2} \cdot R$$

$$\begin{aligned} &= \frac{|(5 + j2.89)|^2}{(63.5)^2} \cdot 26.4 \\ &= 0.218 \text{ MW} \end{aligned}$$

$$\begin{aligned} \Delta Q_L &= \frac{|S|^2}{|V_r|^2} \cdot (X) \\ &= \frac{|(5 + j2.89)|^2}{(63.5)^2} \cdot 33.9 = 0.279 \text{ MVar} \end{aligned}$$

$$\begin{aligned} \text{Power at the sending end of the line } S^1 &= S + \Delta P_L + \Delta Q_L \\ &= (5 + j2.89) + 0.218 + j0.279 \end{aligned}$$

$$= (5.22 + j3.169) \text{ MVA}$$

∴ Total power consumed from the power plant bus bars

$$S_1' = 3S_1$$

$$= 3(5.22 + j2.73) = (15.66 + j 8.19) \text{ MVA}$$

6. A single circuit three phase 220 KV line runs at **no load**. Voltage at the receiving end of the line is 210 KV. Find the **sending end** voltage, if the line has resistance of 20.5 ohms, reactance of **81.3 ohms** and the total susceptance as  $5.45 \times 10^{-4}$  mho. The transmission line is to be represented by  $\pi$  - method.

**Solution:**

$$|V_2| = \frac{210}{\sqrt{3}} = 121.2 \text{ KV}$$

$$Q_c = |V_2|^2 * \frac{WC}{2} = (121.2)^2 * \frac{5.45 * 10^{-4}}{2} = 4 \text{ MVar}$$

$$X = 81.3 \Omega$$

$$\therefore |V_1| = |V_2| - \frac{Q_c}{|V_2|} X$$

$$= 121.2 - \frac{4}{121.2} * 81.3 = 118.51$$

$$\text{Sending end voltage to line} = \sqrt{3} * (V_1)$$

$$= \sqrt{3} * 118.51$$

$$= 205.2 \text{ KV}$$

7. Find the capacity of static VAR compensator to be installed at a bus with  $\pm 5$  voltage fluctuation. The short circuit capacity is 5000 MVA.

**Solution:**

The switching of static shunt compensator,

$\Delta V$  = Voltage fluctuation

$\Delta Q$  = Reactive power variation

S = System short circuit capacity

$$\text{Then } \Delta V = \frac{\Delta Q}{S}$$

$$\Rightarrow \Delta Q = \Delta V * S$$

$$= 0.05 * 5000 = 250 \text{ MVar}$$

The capacity of a static VAR compensator is (+ 250 MVar)



8. A short line having an impedance of  $(1.5 + j2.5) \Omega$  interconnects two plants A and B each plant operates at 11 KV and the two voltage are equal in magnitude and phase. It is proposed to transfer 20 MW power at 0.8 p.f. from plant A to B. Find the voltage boost needed at plant A to achieve this power transfer.

**Solution:**

$$I = \frac{20 \times 10^3}{\sqrt{3} \times 11 \times 0.8} = 1312.2 \angle -36.87^\circ$$

$$= (1049.76 - j787.32)A$$

$$\text{Voltage drop in the inter connector} = (1049.76 - j787.32)(1.5 + j2.5)$$

$$= (3542.94 + j1443.42)V$$

$$\text{The voltage boost needed at station A(plant A) is}$$

$$= (3542.94 + j1443.42) V$$

9. A load bus is composed of induction motor where the nominal reactive power is 1 Pu. The shunt compensation is  $K_{sh}$ . Find the reactive power sensitivity at the bus with respect to change in voltage.

**Solution:**

$$\frac{dQ_{net}}{dv} = 2V - 2V K_{sh}$$

$$V = 1.0 \text{ Pu}$$

$$K_{sh} = 0.8$$

$$\therefore \frac{dQ_{net}}{dv} = (2 \times 1) - (2 \times 1 \times 0.8)$$

$$= 2 - 1.6 = 0.4 \text{ Pu}$$

10. The line is open circuited with a receiving end voltage of 220 KV, find the rms value and phase angle of the following

- The incident and reflected voltages to neutral at the receiving end.
- The incident and reflected voltages to neutral at 200 KM from the receiving end.

$$\alpha = 0.163 \times 10^{-3}$$

$$\beta = 1.068 \times 10^{-3}$$

**Solution:**

- At the receiving end

For open circuit  $I_R = 0$

$$\begin{aligned}\text{Incident voltage} &= \frac{V_R + Z_C I_R}{2} = \frac{V_R + Z_C(0)}{2} = \frac{V_R}{2} \\ &= \frac{220/\sqrt{3}}{2} = 63.51 \angle 0^\circ \text{ KV}\end{aligned}$$

$$\text{Reflected voltage} = \frac{V_R - Z_C I_R}{2} = \frac{V_R}{2} = 63.51 \angle 0^\circ \text{ KV}$$

b) At 200 KM from the receiving end

$$\begin{aligned}\text{incident voltage} &= \frac{V_R}{2} e^{\alpha x} e^{j\beta x} \Big|_{x=200 \text{ km}} \\ &= 63.51 \exp(0.163 * 10^3 * 200) * \exp(j1.068 * 10^{-3} * 200) \\ &= 65.62 \angle 12.2^\circ \text{ kv}\end{aligned}$$

$$\begin{aligned}\text{Reflected voltage} &= \frac{V_R}{2} e^{-\alpha x} e^{-j\beta x} \Big|_{x=200 \text{ km}} \\ &= 63.51 e^{-0.0326} e^{-j0.2135} \\ &= 61.47 \angle -12.2^\circ \text{ KV}\end{aligned}$$

11. A load of 30 MW, 45 MVAR is connected to a line where X to R ratio is 5 and the short circuit capacity of the load bus is 250 MVA. The supply voltage is 11 KV and the load is star connected. Determine the load bus voltage

**Solution:**

$$\text{The load per phase} = 10 + j15 \text{ MVA}$$

$$\text{Short circuit current} = \frac{250}{3} \text{ MVA}$$

$$\text{Voltage line - neutral} = \frac{11}{\sqrt{3}} \text{ KV}$$

$$\text{The equivalent short circuit impedance} = \frac{(11/\sqrt{3})}{250/3} = 0.484 \Omega$$

$$\phi_{sc} = \tan^{-1}(5) = 78.69^\circ$$

$$\therefore R = 0.0949 \Omega$$

$$X = 0.4746 \Omega$$

$$\therefore \left( \frac{11}{\sqrt{3}} \right)^2 = \left( V \cos \phi + \frac{P}{V} R \right)^2 + \left( V \sin \phi + \frac{Q}{V} X \right)^2$$

$$= \left( V * 0.5547 + \frac{10 * 10^6}{V} * 0.0949 \right)^2 + \left( V * 0.832 + \frac{15 * 10^6}{V} * 0.4746 \right)^2$$

$$\frac{121}{3} = \left( 0.5547V + \frac{10}{V} * 0.0949 \right)^2 + \left( 0.832V + \frac{15 * 0.4746}{V} \right)^2$$

$$\Rightarrow 40.33 V^2 = 0.3077 V^4 + 0.9 + 1.053V^2 + 0.69V^4 + 50.68 + 11.846 V^2$$

$$\Rightarrow 0.9977 V^4 - 27.43 V^2 + 51.59 = 0$$

$$\Rightarrow V^4 - 27.5 V^2 + 51.6 = 0$$

$$\Rightarrow V^2 = \frac{27.5 \pm \sqrt{756.25 - 206.32}}{2}$$

$$= \frac{27.5 + 23.45}{2} = 25.47$$

$$V = 5.047 \text{ KV}$$

The drop in volts is =  $6.350 - 5.047 = 1.303 \text{ KV}$

$$\therefore \frac{\Delta V}{V} = \frac{1}{S_{sc}} (P \cos \phi + Q \sin \phi) + j \frac{1}{S_{sc}} (P \sin \phi_{sc} - Q \cos \phi_{sc})$$

$$= \frac{1}{250} (30 * 0.196 + 45 * 0.98) + j \frac{1}{250} (30 * 0.98 - 45 * 0.196)$$

$$= \frac{1}{250} [(5.88 + 44) + j(29.4 - 8.82)]$$

$$= \frac{53.6}{250} = 0.2158$$



## **UNIT-V**

### **POWER SYSTEM OPERATION IN COMPETITIVE ENVIRONMENT**

- 1.1 --INTRODUCTION
- 1.2-- RESTRUCTURING MODELS
- 1.3 --INDEPENDENT SYSTEM OPERATOR (ISO)
- 1.4 --POWER EXCHANGE (PX)
- 1.5-- MARKET OPERATIONS
- 1.6-- MARKET POWER
- 1.7 --STRANDED COSTS
- 1.8-- TRANSMISSION PRICING
- 1.9 --CONGESTION PRICING
- 1.10 --MANAGEMENT OF INTER-ZONAL/INTRAZONAL CONGESTION
- 2.1-- INTRODUCTION
- 2.2-- ELECTRICITY PRICE VOLATILITY
- 2.3 --ELECTRICITY PRICE INDEXES
- 2.4 --CHALLENGES TO ELECTRICITY PRICING
- 2.5-- CONSTRUCTION OF FORWARD PRICECURVES
- 2.6 --SHORT-TERM PRICE FORECASTING

## **1.1 INTRODUCTION**

For many decades, vertically integrated electric utilities monopolized the way they controlled, sold and distributed electricity to customers in their service territories. In this monopoly, each utility managed three main components of the system: generation, transmission and distribution. Analogous to perceived competitions in airline, telephone, and natural gas industries which demonstrated that vertically integrated monopolies could not provide services as efficiently as competitive firms, the electric power industry plans to improve its efficiency by providing a more reliable energy at least cost to customers. A competition is guaranteed by establishing a restructured environment in which customers could choose to buy from different suppliers and change suppliers as they wish in order to pay market-based rates.

To implement competition, vertically integrated utilities are required to unbundle their retail services into generation, transmission and distribution; generation utilities will no longer have a monopoly, small businesses will be free to sign contract for buying power from cheaper sources, and utilities will be obligated to deliver or wheel power over existing lines for a fee that is the same as the cost (non-discriminatory) delivering the utility's own power without power production costs. The vertically integrated system is steadily restructuring to a more market based system in which competition will replace the role of regulation in setting the price of electric power.

## **1.2 RESTRUCTURING MODELS**

Three major models are being discussed as alternatives to the current vertically integrated monopoly. The three models are:

- a) PoolCo Model
- b) Bilateral Contracts Model
- c) Hybrid Model

Elements of a certain electric power industry define the nature of competition and models or institutions that support the competition process. In adopting a model, the following issues are being debated regularly:

- Who will maintain the control of transmission grid?
- What types of transactions are allowed?
- What level of competition does a system warrant?

A PoolCo is defined as a centralized marketplace that clears the market for buyers and sellers where electric power sellers/buyers submit bids and prices into the pool for the amounts of energy that they are willing to sell/buy. The ISO or similar entities (e.g. PX) will forecast the demand for the following day and receive bids that will satisfy the demand at the lowest cost and prices for electricity on the basis of the most expensive generator in operation (marginal generator). On the other hand, in the second model, bilateral trades are negotiable and terms and conditions of contracts are set by traders without interference with system operators.

### **1.2.1 PoolCo Model:**

The PoolCo model is comprised of competitive power providers as obligatory members of an independently owned regional power pool, vertically integrated distribution companies, vertically integrated transmission companies and a single and separate entity responsible for: establishing bidding procedures, scheduling and dispatching generation resources, acquiring necessary ancillary services to assure system reliability, administering the settlements process and ensuring non-discriminatory access to the transmission grid. PoolCo does not own any generation or transmission components and centrally dispatches all generating units within the service jurisdiction of the pool. PoolCo controls the maintenance of transmission grid and encourages an efficient operation by assessing non-discriminatory fees to generators and distributors to cover its operating expenses.

In a PoolCo, sellers and buyers submit their bids to inject power into and out of the pool. Sellers compete for the right to inject power into the grid, not for specific customers. If a power provider bids too high, it may not be able to sell power. On the other hand, buyers compete for buying power and if their bids are too low, they may not be getting any power. In this market, low cost generators would essentially be rewarded. Power pools would implement the economic dispatch and produce a single (spot) price for electricity, giving participants a clear signal for consumption and investment decisions. Winning bidders are paid the spot price that is equal to the highest bid of the winners. Since the spot price may exceed the actual running of selected bidders, bidders are encouraged to expand their market share which will force high cost generators to exit the market. Market dynamics will drive the spot price to a competitive level that is equal to the marginal cost of most efficient firms.

To give the reader an idea on how price signals could play an important role in a restructured environment, we consider the following example.

### Example 1.1: Impact of Price Signals on Demand Consumption

Figures 1.1 and 1.2 show the idea of price signals and their impact on consumption behavior. Let's assume that an Industrial Customer (IndustCo) consumption pattern in a vertically integrated electric industry, in a certain day, would look like that shown in Figure 1.1. In this figure, the price of electricity in Rs/KWh is fixed, and the IndustCo has no incentives to change its consumption pattern. While in a competition-based market, when the IndustCo receives a real time price signal, it could change its consumption pattern in response to the price (See Figure 1.2). In Figure 1.2, the IndustCo reduces its usage at times high prices. As the price jumps from 6Rs/KWh at 5 a.m. to 8Rs/KWh at 9 a.m. the IndustCo starts decreasing its consumption. As the price continues to increase from 8Rs/KWh at 9 a.m. to 16Rs/KWh at 6 p.m. the IndustCo continues decreasing its consumption. When the price decreases after 6 p.m. the IndustCo increases its usage of electricity.

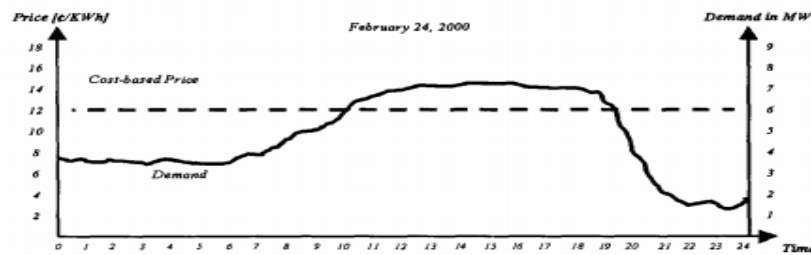


Figure 1.1 Behavior of a Demand in a Vertically Integrated Power Market

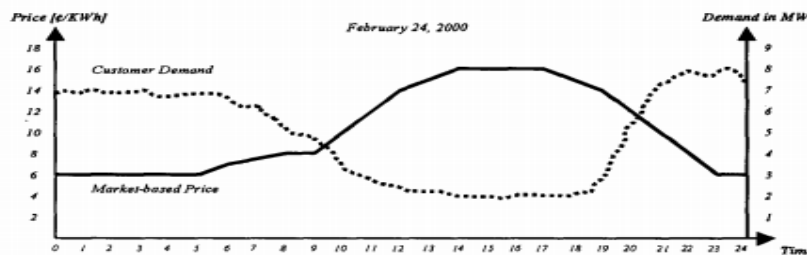


Figure 1.2 Response of a Demand to Price Signals

Although buyers and sellers in a PoolCo are prevented from making individual contracts for power, participants may hold optional financial instruments called Contracts for Differences (CFDs). These contracts are long-term price hedging bilateral contracts between generators and distribution utilities or retail customers. These contracts allow a physical dispatch of individual generating units by their owners and allow consumers to establish long-term prices. When used, a power seller is paid a fixed amount over time that is a combination of short-term market price and an adjustment for the difference. CFDs are established as a mechanism to stabilize power costs to customers and revenues for generators.



These contracts are suggested due to the fact that the spot price set by PoolCo fluctuates over a wide range and is difficult to forecast over long periods. Using CFDs, any differences between the spot price and the contract price would be offset by cash payments by generators to customers; in other words, by holding these contracts, customers gain protection against unexpected spot price increases and generators could obtain a greater revenue stability.

### **1.2.2 Bilateral Contracts (Direct Access) Model.**

The Bilateral Contracts model has two main characteristics that would distinguish it from the PoolCo model. These two characteristics are: the ISO's role is more limited; and buyers and sellers could negotiate directly in the marketplace. In this model, small customers' aggregation is essential to ensure that they would benefit from competition.

This model permits direct contracts between customers and generators without entering into pooling arrangements. By establishing non-discriminatory access and pricing rules for transmission and distribution systems, direct sales of power over a utility's transmission and distribution systems are guaranteed. Wholesale suppliers would pay transmission charges to a transmission company to acquire access to the transmission grid and pay similar charges to a distribution company to acquire access to the local distribution grid. In this model, a distribution company may function as an aggregator for a large number of retail customers in supplying a long-term capacity. Also, the generation portion of a former integrated utility may function as a supplier or other independent generating companies, and transmission system would serve as a common carrier to contracted parties that would permit mutual benefits and customer's choice. Any two contracted parties would agree on contract terms such as price, quantity and locations, and generation providers would inform the ISO on how its hourly generators would be dispatched.

### **1.2.3 Hybrid Model:**

The hybrid model combines various features of the previous two models. The hybrid model differs from the PoolCo model as utilizing the PX is not obligatory and customers are allowed to sign bilateral contracts and choose suppliers from the pool. The pool would serve all participants (buyers and sellers) who choose not to sign bilateral contracts. The California model is an example of the hybrid model. This structure has advantages over a mandatory pool as it provides end-users with the maximum flexibility to purchase from either the pool or directly from suppliers. A customer who would choose a PX option with CFDs could acquire the economic equivalent of bilateral contracts.

The existence of the pool can efficiently identify individual customer's energy requirements and simplify the balancing process of energy supply. The hybrid model would enable market participants to choose between the two options based on provided prices and services. The hybrid model is very costly to set up because of separate entities required for operating the PX and the transmission system. In the following, we learn more about the functions of an ISO in a restructured power system.

## **1.3 INDEPENDENT SYSTEM OPERATOR (ISO)**

### **1.3.1 Background:**

In a vertically integrated monopoly, utilities created regional centrally dispatched power pools to coordinate the operation and planning of generation and transmission among their members in order to improve operating efficiencies by selecting the least-cost mix of generating and transmission capacity, coordinating maintenance of units, and sharing operating reserve requirements and thus lowering the cost to end-use electricity customers. The centrally dispatched power pools are classified as:

- **Tight power pools:** they have customarily functioned as control areas for their members; perform functions such as unit commitment, dispatch and transaction scheduling services. Examples of this kind of pools are New York Power Pool (NYPP), New England Power Pool (NEPOOL), Pennsylvania, New Jersey, Maryland (PJM) Interconnection, Colorado Power Pool and Texas Municipal Power Pool.
- **Loose power pools:** they have generally had more limited roles, and in contrast to tight pools, have a low level of coordination in operation and planning. The most significant role of these

pools has provided support in emergency conditions. Loose power pools did not provide control area services.

- **Affiliate power pools:** in this kind of pools, generated power which was owned by the various companies was dispatched as a single utility. Pools have had extensive agreements on governing the cost of generation services and use of transmission systems.

Power pools controlling access to regional transmission systems made it difficult for non-members to use pool members' transmission facilities by establishing complex operating rules and financial arrangements. Also, restrictive membership and governance of pools were practiced occasionally in a way that large utilities prevented changes in policies and rules of the pool which led to closing pool membership to outsiders. Unfair industry practices generally impacted the growth of a competitive generation market and were motive forces for the FERC to order transmission owners to provide other parties an open access to transmission grids.

A competitive generation market and retail direct access necessitated an independent operational control of the grid. However, the independent operation of the grid was not guaranteed without an independent entity, the so-called Independent System Operator (ISO). An ISO is independent of individual market participants such as transmission owners, generators, distribution companies and end-users. The basic purpose of an ISO is to ensure a fair and non-discriminatory access to transmission services and ancillary services, maintain the real-time operation of the system and facilitate its reliability requirements.

### **1.3.2 The Role of ISO:**

The primary objective of the ISO is not dispatching or re-dispatching generation, but matching electricity supply with demand as necessary to ensure reliability. ISO should control generation to the extent necessary to maintain reliability and optimize transmission efficiency. The ISO would continually evaluate the condition of transmission system and either approve or deny requests for transmission service.

In its Order No. 888, FERC developed eleven principles as guidelines to the electric industry restructuring to form a properly constituted ISO, through which public utilities could comply with FERC's non-discriminatory transmission tariff requirements. The eleven principles for ISOs are:

- 1) The ISO's governance should be structured in a fair and non-discriminatory manner.
- 2) An ISO and its employees should have no financial interest in the economic performance of any power market participant. An ISO should adopt and enforce strict conflict of interest standards.
- 3) An ISO should provide open access to the transmission system and all services under its control at non-pancaked rates pursuant to a single, unbundled, grid-wide tariff that applies to all eligible users in a non-discriminatory manner.
- 4) An ISO should have the primary responsibility in ensuring short-term reliability of grid operations. Its role in this responsibility should be well defined and comply with applicable standards set by NERC and the regional reliability council.
- 5) An ISO should have control over the operation of interconnected transmission facilities within its region.
- 6) An ISO should identify constraints on the system and be able take operational actions to relieve those constraints within the trading rules established by the governing body. These rules should promote efficient trading.
- 7) An ISO should have appropriate incentives for efficient management and administration and should procure services needed for such management and administration in an open market.
- 8) An ISO's transmission and ancillary services pricing policies should promote the efficient use of and investment in generation, transmission, and consumption. An ISO or a Regional Transmission Group (RTG) of which the ISO is a member should conduct such studies as may be necessary to identify operational problems or appropriate expansions.

- 9) An ISO should make transmission system information publicly available on a timely basis via an electronic information network consistent with the Commission's requirements.
- 10) An ISO should develop mechanisms to coordinate its activities with neighboring control areas.
- 11) An ISO should establish an Alternative Dispute Resolution (ADR) process to resolve disputes in the first instance.

According to the FERC Order 888, the ISO is authorized to maintain transmission system reliability in real-time. To comply with the FERC Order 888, each ISO may take one of the following structures:

The first structure is mainly concerned with maintaining the transmission reliability by operating the power market to the extent that the ISO would schedule transfers in a constrained transmission system. An example of this proposal is the Midwest ISO.

The second proposal for an ISO includes a PX that is integral to the ISO's operation. In some proposals such as those of the UK and the PJM interconnection, the PX would function within the same organization and under the control of the ISO; the ISO is responsible for dispatching all generators and would set the price of energy at each hour based on the highest price bid in the market.

In its Order 888, FERC also defined six ancillary services that must be provided by or made available through transmission providers. These ancillary services include:

- 1) Scheduling, Control and Dispatch Services
- 2) Reactive Supply and Voltage Control
- 3) Regulation and Frequency Response Services
- 4) Energy Imbalance Service
- 5) Operating Reserve, Spinning and Supplemental Reserve Services
- 6) Transmission Constraint Mitigation

As Order 888 implies, transmission customers may self-provide these services or buy them through one of the following methods.

- (i) Providers of these services advertise their availability via the OASIS or commercial exchanges
- (ii) The ISO provides these services in real-time and charges transmission users.

To make these services available, the ISO contracts with service providers so that the services are available under the ISO's request. When a service provider is called by the ISO, the provider is paid extra to cover its operating costs. Capacity resources are contracted seasonally by the ISO and providers send their bids to an auction operated by the ISO. The ISO chooses successful providers based on a least-cost bid basis. When determining the winners, the ISO takes into account factors such as time and locational constraints and the expected use of resources. If the ISO finds that spot market services are less expensive than contracted ones, the ISO exercises its authority by acquiring these services from the energy spot market.

The following example illustrates the role of ISO in providing operating reserves.

#### **Example 1.2: (Operating Reserves)**

Assume that a hydroelectric generator, which has a 30 MW capacity and can be brought up and running in 4 minutes, offers its 30 MW capacity in the operating reserve market. It submits a reserve price offer of \$2.5/MW for each hour of the entire next day, along with an energy offer price of \$42.5/MWh. Also, assume that there are two customers, C1 and C2 that can cut back quickly their usage of electricity. C1 can cut up to 15 MW in 5 minutes, so it decides to submit a bid to the operating reserve market of 15 MW at \$1.5/MW for each hour of the entire next day and a maximum energy bid price of \$55/MWh. C2 can cut up to 10 MW in 5 minutes, so it decides to submit a bid to the operating reserve market of 10 MW at \$2/MW for each hour of the entire next day and a maximum energy bid price of \$70/MWh.

The ISO anticipates that it will need a large operating reserve, say 55 MW, so it accepts the generator's offer and the customers' bids. Assume that the MCP of the operating reserves is \$2.5/MW/h. Each winning participant in the operating reserve market will be paid the MCP to stand by in case of

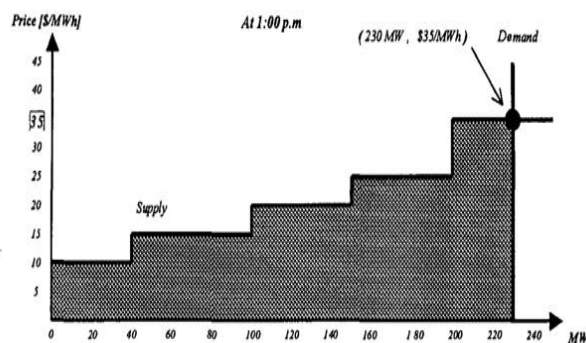


contingency. The generator will be paid \$1800 (or  $\$2.5/\text{MW}/\text{h} \times 30\text{MW} \times 24\text{h}$ ), C will be paid \$900 (or  $\$2.5/\text{MW}/\text{h} \times 15\text{MW} \times 24\text{h}$ ), and C2 will be paid \$600 (or  $\$2.5/\text{MW}/\text{h} \times 10\text{MW} \times 24\text{h}$ ).

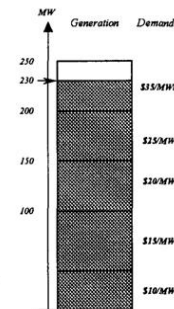
In the next day, say at 1:00 p.m., let's assume that supply and demand equilibrium is initially established at 230 MW (in the spot market). The spot market price of electricity at this point is \$35/MWh.

The net supply curve of all generating units is as shown in Figure 1.3. The supply segment of \$20/MWh is offered by a single generating unit that has a capacity of 50 MW. As segment) that is dispatched last (in of its total capacity of 50 MW, and shown in Figure 1.3, the unit (supply the spot market) is producing 30 MW has available capacity of 20 MW.

Now suppose the generating unit that is producing 50 MW at \$20/MWh tripped out due to severe weather conditions at 1:10 p.m. which disconnected the line joining this generating unit to the system. The supply curve after the unit outage is shown in Figure 1.4. At 1:10 p.m., demand is still 230 MW, and the supply has a shortage of 50 MW. At this point, the ISO has to replace the sudden loss of capacity in the remainder of the hour (from 1:10-2:00 p.m.), before more capacity can be dispatched in the spot market. So the ISO has to reduce the demand if possible or/and provide the required power from operating reserves.

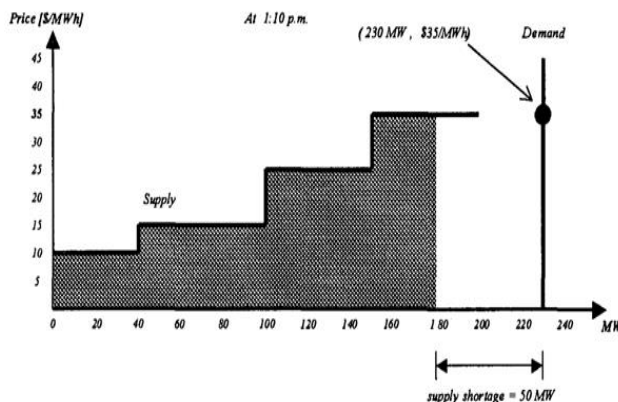


1.3.a Supply and Demand Curves of Initial Situation

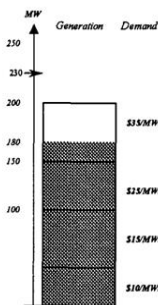


1.3.b Total Production and Production of each Generating Unit of the Initial Situation

Figure 1.3 Initial Situation



1.4.a Supply and Demand Curves after the Unit Outage



1.4.b Total Production and Production of each Generating Unit after the Outage

Figure 1.4 Situation after the Unit Outage

The ISO dispatches the 30 MW generator (see Figure 1.5), and the generator is paid \$42.5/MWh. To restore the balance, the ISO also dispatches off 15 MW from C1 and 5 MW from C2 (See Figure 1.6). The ISO pays \$20/MWh (or  $\$55/\text{MWh} - \$35/\text{MWh}$ ) to C~ and \$35/MWh (or  $\$70/\text{MWh} - \$35/\text{MWh}$ ) to C2

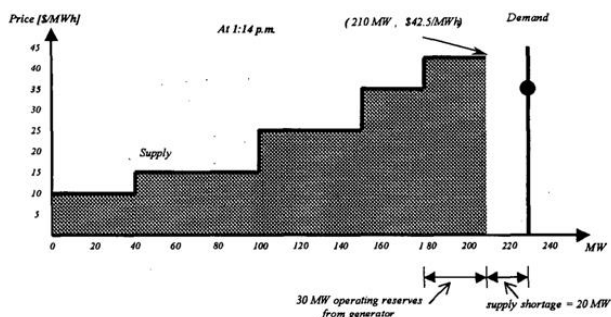
for the electricity that it has dispatched off. The payments to the three participants (generator, C1 and C2) continue until replacement energy can be provided from the spot market.

#### 1.4 POWER EXCHANGE (PX)

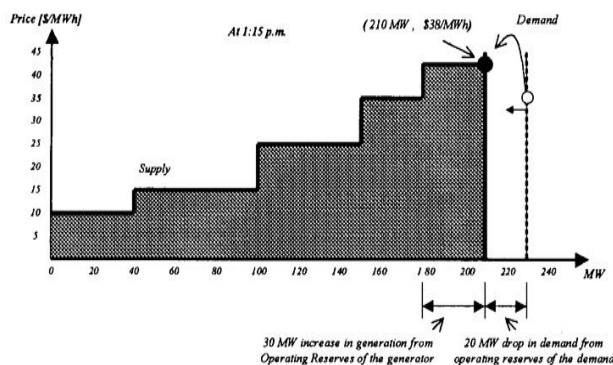
Even though short-term and long-term financial energy transactions could be in bilateral forms in the electricity industry where contracted parties agree individually for certain terms such as price, availability and quality of products, industry restructuring proposals have concluded the necessity of creating a new marketplace to trade energy and other services in a competitive manner. This marketplace is termed Power Exchange (PX) or, as sometimes called, spot price pool. This marketplace permits different participants to sell and buy energy and other services in a competitive way based on quantity bids and prices. Participants include utilities, power marketers, brokers, load aggregators, retailers, large industrial customers and co-generators.

PX is a new independent, non-government and non-profit entity which accept schedules for loads and generation resources. It provides a competitive marketplace by running an electronic auction where market participants buy and sell electricity and can do business quickly and easily. Through an electronic auction, PX establishes an MCP for each hour of the following day for trades between buyers (demands) and sellers (supplies). In this marketplace, PX does not deal with small consumers. Add to that, PX manages settlement and credit arrangements for scheduling and balancing of loads and generation resources. As a main objective in its work, PX guarantees an equal and non-discriminatory access and competitive opportunity to all participants. It is claimed that participants entering the PX get more cost-effectiveness by removing the complexities of arranging generation, transmission and energy purchases.

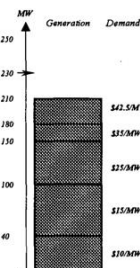
In general, the PX includes a day-ahead market and an hour-ahead market. Here we discuss these markets in general, and later we elaborate on them when we discuss some market models in the United States.



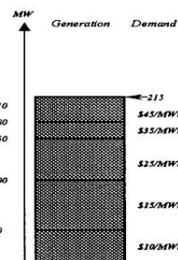
1.5.a Supply and Demand Curves after the ISO dispatches the 30 MW generator



1.6.a Supply and Demand Curves after the ISO dispatches off the 20 MW



1.5.b Total Production and Production of each Generating Unit after the ISO dispatches the 30 MW generator



1.6.b Total Production and Production of each Generating Unit after the ISO dispatches off the 20 MW

Figure 1.6 Situation after Drop of  $C_1$  and  $C_2$  Demand

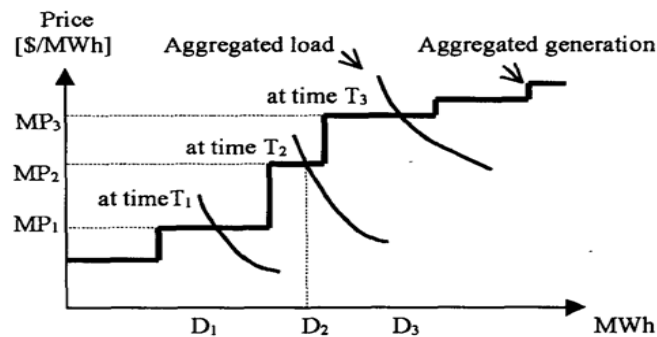
#### 1.4.1 Market Clearing Price (MCP):

PX accepts supply and demand bids to determine a MCP for each of the 24 periods in the trading day. Computers aggregate all valid (approved) supply bids and demand bids into an energy supply curve and an energy demand curve. MCP is determined at the intersection of the two curves and all trades are executed at the MCP, in other words, the MCP is the balance price at the market equilibrium for the aggregated supply and demand graphs. Figure 1.7 shows the determination of MCPs for certain hours when demand ( $D_i$ ) varies. Generators are encouraged to bid according to their operating costs because bidding lower would lead to financial losses if MCP is lower than the operating cost and bidding higher could cause units to run less frequently or not run at all.

## 1.5 MARKET OPERATIONS:

### 1.5.1 Day-Ahead and Hour-Ahead Markets:

In the day-ahead market and for each hour of the 24-hour scheduling day, sellers bid a schedule of supply at various prices, buyers bid a schedule of demand at various prices, and MCP is determined for each hour. Then, sellers specify the resources for the sold power, and buyers specify the delivery points for the purchased power. PX schedules supply and demand with the ISO for delivery. Supply and demand are adjusted to account for congestion and ancillary services and then PX finalizes the schedules.



**Figure 1.7** Process of Determining MCP in PoolCo

The hour-ahead market is similar to day-ahead, except trades are for 1 hour, and the available transfer capability (ATC) is reduced to include day-ahead trades, and bids are not iterative in this market. Once the MCP is determined in the PX, market participants submit additional data to the PX. The data would include individual schedules by generating unit; take out point for demand, adjustment bids for congestion management and ancillary service bids. After this stage, the ISO and the PX know the injection points of individual generating units to the transmission system. A schedule may include imports and/or exports. To account for transmission losses, generator's schedules are adjusted where real losses are only known after all metered data are processed.

### 1.5.2 Elastic and Inelastic Markets.

An inelastic market does not provide sufficient signals or incentives to a consumer to adjust its demand in response to the price, i.e., the consumer does not have any motivation to adjust its demand for electrical energy to adapt to market conditions. In a market that has a demand; MCP for energy is determined by the price structure of supply offers. The concept of inelastic demand is directly related to the concept of firm load, which formed the basis of the electricity industry for many decades before the introduction of open access and energy markets. Customers use the concept of elastic demand when they are exposed to and aware of the price of energy and arrange their affairs in such a fashion as to reduce their demand as the price of the next available offer exceeds a certain level.

The following example illustrates how the elasticity of demand in a market has some serious impacts on the energy market and the power system itself, and demonstrates how the pool price cap (price ceiling) energy markets with inelastic and elastic demand may play important role. The example also illustrates the need for capacity reserves.

## 1.6 MARKET POWER:



One of the main anti-competitive practices or difficulties that may prevent competition in the electric power industry, especially in generation, is market power. When an owner of a generation facility in a restructured industry is able to exert significant influences (monopoly) on pricing or availability of electricity, we say that market power exists, and if so it prevents both competition and customer choice. Market power may be defined as owning the ability by a seller, or a group of sellers, to drive price over a competitive level, control the total output or exclude competitors from a relevant market for a significant period of time. Other than price, any entity that exercises market power would reduce competition in power production, service quality and technological innovation. The net result of practicing market power is a transfer of wealth from buyers to sellers or a misallocation of resources.

There are two types of market power:

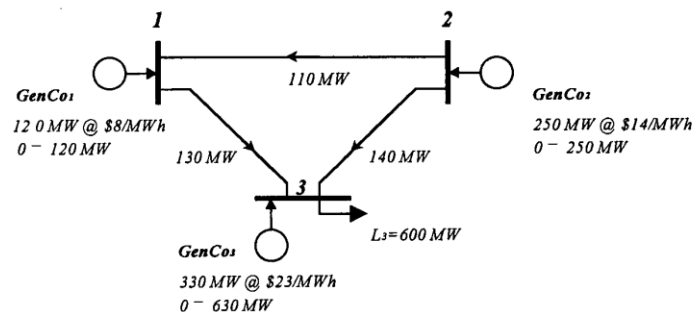
- 1-Vertical Market Power:** It arises from a single-firm's or affiliate's ownership of two or more steps in a production and market delivery process where one of the steps provides the firm with a control of a bottleneck in the process. The bottleneck facility is a point on the system, such as a transmission line, through which electricity must pass to get to its intended buyers. A control of a bottleneck process enables the firm to give preference to itself or its affiliate over competitive firms.
- 2- Horizontal Market Power:** It is the ability of a dominant firm or group of firms to control production to restrict output and thereby raise prices. It arises from a firm's local control or ownership concentration of a single process step in productive assets within a defined market area. If such concentration is sufficient with respect to certain other market conditions, the firm can influence the supply-demand equilibrium, and hence prices, simply by withholding production. This type of market power cannot be resolved by the ISO.

Concentration in a market measures the market dominance (degree of monopoly experienced by a firm in a competitive market) using market share data, i.e., how many firms exist in the market, and what are their relative sizes determine the market dominance.

**Example 1.6: (How is Market Power Exercised?)**

(A) Exercising Market Power when a Power Supply has a Large Market Share,

Figure 1.14 shows a 3-bus system with three generating companies, one at each bus. The three generating companies are GenCo<sub>1</sub> with a maximum capacity of 120 MW, GenCo<sub>2</sub> with a maximum capacity of 250 MW, and GenCo<sub>3</sub> with a maximum capacity of 630 MW.



**Figure 1.14** Exercising Market Power when a Power Supply has Large Market Share

The HHI for this situation is

$$HHI = \sum_i^3 S_i^2 = \left(\frac{120}{1000}\right)^2 + \left(\frac{250}{1000}\right)^2 + \left(\frac{630}{1000}\right)^2$$

$$= 0.0144 + 0.0625 + 0.3969 = 0.4738 \text{ (in per unit basis)}$$

or,

$$HHI = \sum_i^3 S_i^2 = \left(\frac{120}{1000} \times 100\right)^2 + \left(\frac{250}{1000} \times 100\right)^2 + \left(\frac{630}{1000} \times 100\right)^2$$

$$= 144 + 625 + 3969 = 4738 \text{ (in percent basis)}$$

Note that GenCo<sub>3</sub> owns the maximum share of the total generation capacity (=630MW/1000MW=63%). By ignoring the limitation transmission lines, GenCo<sub>3</sub> monopolizes the market, because L<sub>3</sub> needs much more than the total capacity of the other cheaper resources (GenCo<sub>1</sub> can generate up to 120 MW at \$8/MWh and GenCo<sub>2</sub> can generate up to 250 MW at \$14/MWh). It means when GenCo<sub>3</sub> wants to exercise its market power, it can ask for any price for its electric power production to fulfill L<sub>3</sub>'s need.

#### (B) Exercising Market Power when Transmission System is Congested

Figure 1.15 shows a 3-bus system with three generating companies, one at each bus. The three generating companies are GenCo<sub>1</sub> with a maximum capacity of 250 MW, GenCo<sub>2</sub> with a maximum capacity of 350 MW, and GenCo<sub>3</sub> with a maximum capacity of 400 MW. The HHI for this situation is

$$HHI = \sum_i^3 S_i^2$$

$$= \left(\frac{250}{1000}\right)^2 + \left(\frac{350}{1000}\right)^2 + \left(\frac{400}{1000}\right)^2$$

$$= 0.0625 + 0.1225 + 0.1600 = 0.345 \text{ (in per unit basis)}$$

or,

$$HHI = \sum_i^3 S_i^2 = \left(\frac{250}{1000} \times 100\right)^2 + \left(\frac{350}{1000} \times 100\right)^2 + \left(\frac{400}{1000} \times 100\right)^2$$

$$= 625 + 1225 + 1600 = 3450 \text{ (in percent basis)}$$

In this case GenCo<sub>1</sub>, GenCo<sub>2</sub> and GenCo<sub>3</sub> own, respectively, 25%, 35%, and 40% of the total generation capacity. Transmission line limits are imposed on the system in this case, as shown in Figure 1.15. Even though the cheapest resources (GenCo<sub>1</sub> and GenCo<sub>2</sub>) have a total capacity of 600 MW, which is adequate to cover the 600 MW required by L<sub>3</sub>, the limitations of the transmission lines do not permit L<sub>3</sub> to use GenCo<sub>1</sub> and GenCo<sub>2</sub>. This situation may lead to exercising market power by GenCo<sub>3</sub> by imposing a higher than competitive price.

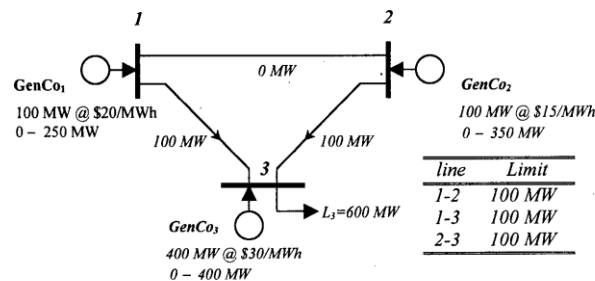


Figure 1.15 Exercising Market Power when Transmission is Congested

### 1.7 STRANDED COSTS:

A major and a debatable issue associated with the electric utility restructuring is the issue of stranded costs; how to be determined, how to be recovered and who pays for recovery. Stranded cost is a terminology created under restructuring process. Multiplicity of definitions and interpretations of stranded costs confused people working on restructuring, but in general this term refers to the difference

between costs that are expected to be recovered under the rate regulation and those recoverable in a competitive market.

In a vertically integrated monopoly, utilities are used to cover their costs of doing business in rates charged to customers. Costs include operating costs and invested capital costs, where utilities cover these costs and considerable returns on their capitals through rates imposed on customers. But when restructuring is proposed to open market-based competition, and due to the fact that market-based prices are uncertain and sometimes less than vertically integrated rates, financial obligations of vertically integrated utilities may become unrecoverable in a competitive market and the level of revenue earned by a utility may no longer be adequate to cover its costs. If market prices are lower than vertically integrated rates, as many expect, utilities could be faced with investments that are unrecoverable in the competitive market.

Stranded costs still need a more clear definition (what costs should be strandable? what costs are unrecoverable? and to what extent (totally or partially) should be recovered?) and quantification. On the other hand, the duration of recovery or who will pay for recovery is not clear yet and varies from model to model in the United States. In this regard, the big question is whether different participants should pay for uneconomical previous investments.

### **1.8 TRANSMISSION PRICING**

FERC recognized that transmission grid is the key issue to competition, and issued guidelines to price the transmission. The guidelines are summarized such that the transmission pricing would:

- (i) Meet traditional revenue requirements of transmission owners
- (ii) Reflect comparability: i.e. a transmission owner would charge itself on the same basis that it would charge others for the same service
- (iii) Promote economic efficiency
- (iv) Promote fairness
- (v) Be practical

Even though transmission costs are small as compared to power production expenses and represent a small percent of major investor owned utilities operating expenses, a transmission system is the most important key to competition because it would provide price signals that can create efficiencies in the power generation market. The true price signals are used as criteria for adding transmission capacity, generation capacity, or future loads. Adding transmission capacity to relieve transmission constraints could allow high-cost generation to be replaced by less expensive generation, which would result in additional savings to consumers.

#### **1.8.1 Contract Path Method:**

It has been used between transacted parties to price transmission where power flows are assumed to flow over a predefined path(s). Despite its ease, this technique was claimed to be a bad implementation of true transmission pricing as power flows would very seldom correspond to predefined paths. Physically, electrons could flow in a network over parallel paths<sup>25</sup> (loop flows) owned several utilities that may not be on the contract path. As a result, transmission owners may not be compensated for the actual use of their facilities. Added to parallel flows, the pancaking<sup>26</sup> of transmission rates is another shortcoming of this method.

As a solution to the pancaking effect, zonal pricing schemes have been proposed by most ISOs. Using a zonal scheme, the ISO-controlled transmission system is divided into zones and a transmission user would pay rates based on energy prices in zones where power is injected or withdrawn. When the zonal approach is used, rates are calculated regardless of paths between the two zones or how many other zones are crossed.

#### **1.8.2 The MW-Mile Method:**

We illustrate this method by an example.

Several ISOs are using a MW-Mile approach to price transmission. The MW-Mile rate is basically based on the distance between transacted parties (from the generating source to the load) and

flow in each line resulted from the transaction. This approach takes into account parallel power flows and eliminates the previous problem that transmission owners were not compensated for using their facilities. This approach does not give credit for counter-flows on transmission lines. The method is complicated because every change in transmission lines or transmission equipment requires a recalculation of flows and charges in all lines.

### Example 1.8: (MW-Mile Charges)

Figure 1.19 illustrates a 3-bus system and a situation of five transactions between generating units and loads. Table 1.8 shows the system data. Let variable  $L_{n,j}$  refer to a load that exists at bus  $n$  and supplied by generator  $j$ . Assume that a load of 150 MW exists at bus 2 which is supplied by generator 1 ( $L_{2,1}=60$  MW) and generator 3 ( $L_{2,3}=90$  MW). Also another load of 600 MW exists at bus 3, which is supplied by the three generators, 240 from generator 1, 200 MW from Generator 2, and 160 MW from generator 3 (or  $L_{3,1}=240$  MW,  $L_{3,2}=200$  MW,  $L_{3,3}=160$  MW).

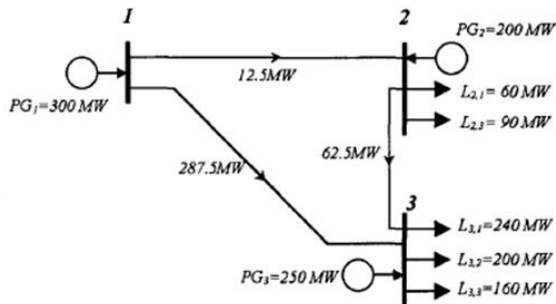


Figure 1.19 3-bus System with Five Transactions

Table 1.8 System Data of the Example

Line	Resistance [ $\Omega$ ]	Reactance [ $\Omega$ ]	Length [Mile]	R [\$/Mile]
1-2	0.0	0.30	20.0	5.0
1-3	0.0	0.10	10.0	23.0
2-3	0.0	0.40	40.0	10.0

We assume the system is lossless and we use dc-load flow equations. Furthermore, bus 3 is the reference bus ( $\delta_3 = 0.0$ ) and  $P_i$  is the net injection (net generation - net load) at bus  $i$ . For this system, we find voltage angles as follows:

$$\begin{bmatrix} 1/0.3 + 1/0.1 & -1/0.3 \\ -1/0.3 & 1/0.3 + 1/0.4 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 13.3333 & -3.3333 \\ -3.3333 & 5.8333 \end{bmatrix}^{-1} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{66.6666} \begin{bmatrix} 5.8333 & 3.3333 \\ 3.3333 & 13.3333 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad (a)$$

$$= \begin{bmatrix} 0.0875 & 0.0500 \\ 0.0500 & 0.2000 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$



Let  $x_{mn}$  refer to the reactance of a line connecting buses  $m$  and  $n$ , and  $f_{mn}$  refer to the flow from  $m$  to  $n$ . Then,

$$f_{mn} = \frac{\delta_m - \delta_n}{x_{mn}} \quad (b)$$

After finding voltage angles, we calculate line flows using Equation b. The net line flows are shown in Figure 1.19.

If  $f_{m-n,j}$  is the loading of line  $m-n$  due to transaction  $j$ ,  $D_{m-n}$  is the length of line  $m-n$  in miles, and  $R_{m-n}$  is the required revenue per unit length of line  $m-n$  (\$/mile), the MW-Mile method uses the following equation to find charges for line  $m-n$  corresponding to transaction  $j$  (i.e.  $C_{m-n,j}$ ):

$$C_{m-n,j} = \frac{f_{m-n,j} D_{m-n} R_{m-n}}{f_{m-n}} = \frac{f_{m-n,j} Z_{m-n}}{f_{m-n}}$$

where  $z_{m-n}$  is the required revenue of line  $m-n$  in \$, i.e.,  
 $z_{m-n} = D_{m-n} R_{m-n}$ .

Line flows due to each transaction are shown in Figures 1.20–1.22 and detailed in Table 1.9. Note that the total flow in each line given in Table 1.9 is equal to the total flow in each line given in Figure 1.19. Table 1.9 shows MW-Mile charges for each transaction's contributions to line flows as well as the total charges paid by each transaction for its contributions. The table also shows charges paid by all transactions in each line.

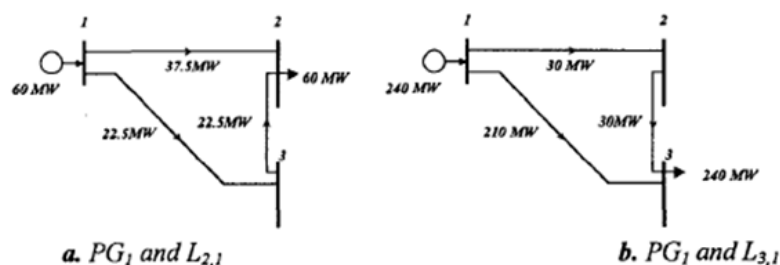


Figure 1.20 Transactions of PG<sub>1</sub>

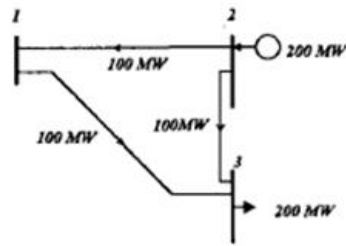


Figure 1.21 Transaction of  $PG_2$  and  $L_{3,2}$

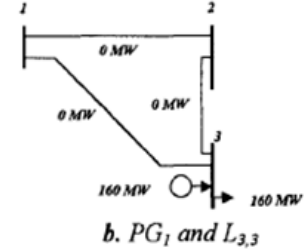
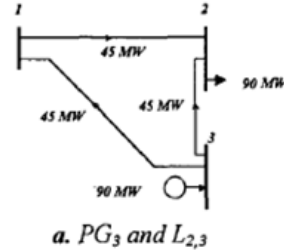


Figure 1.22 Transactions of  $PG_3$

Table 1.9 Impact of each Transaction on Line Flows

Transaction $j$	$f_{1-2,j}$ (1→2)	Cost [\$]	$f_{1-3,j}$ (1→3)	Cost [\$]	$f_{2-3,j}$ (2→3)	Cost [\$]	Total [\$]
1. 60 MW : $PG_1 \rightarrow L_{2,1}$	37.5	300.	22.5	18.	-22.5	144.	199.5
2. 240 MW : $PG_1 \rightarrow L_{3,1}$	30.0	240.	210.	168	30.	192.	600.0
3. 200 MW : $PG_2 \rightarrow L_{3,2}$	-100.	800.	100.	80	100.	640.	1,520.
4. 90 MW : $PG_3 \rightarrow L_{2,3}$	45.0	360.	-45.0	36	-45.	288.	684.0
5. 160 MW : $PG_3 \rightarrow L_{3,3}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	12.5 MW	\$1,700	287.5 MW	\$302	62.5 MW	1,264.	\$3,003.

## 1.9 CONGESTION PRICING:

The condition where overloads in transmission lines or transformers occur is called congestion. Congestion could prevent system operators from dispatching additional power from a specific generator. Congestion could be caused for various reasons, such as transmission line outages, generator outages, changes in energy demand and uncoordinated transactions. Congestion may result in preventing new contracts, infeasibility in existing and new contracts, additional outages, monopoly of prices in some regions of power systems and damages to system components. Congestion may be prevented to some extent (preventive actions) by means of reservations, rights and congestion pricing. Also, congestion can be corrected by applying controls (corrective actions) such as phase shifters, tap transformers, reactive power control, re-dispatch of generation and curtailment of loads.

FERC has set guidelines for a workable market approach to manage congestion, which are:

- (1) The approach should establish clear and trade able rights for transmission usage,
- (2) The approach should promote efficient regional dispatch,
- (3) The approach should support the emergence of secondary markets for transmission rights,
- (4) The approach should provide market participants with the opportunity to hedge locational differences in energy prices,
- (5) Congestion pricing method should seek to ensure that the generators that are dispatched in the presence of transmission constraints are those that can serve system loads at least cost, and
- (6) The method should ensure that the transmission capacity is used by market participants who value that use most highly.

As such, FERC declares that some approaches appear to have more advantages than others. Even though LMP scan be costly and difficult to implement, especially by entities that have not previously operated as tight power pools, FERC suggests that markets that are based on LMP and financial rights for

firm transmission service will provide an efficient congestion management framework, and this is due to the following facts:

**I-** LMP assigns congestion charges directly to transmission customers in a fashion that agrees with each customer's actual use of the system and the actual dispatch that its transactions cause.

**II-** LMP facilitates the creation of financial transmission rights, which enable customers to pay known transmission rates and to hedge against congestion charges.

**III-** Financial rights entitles their holders to receive a share of congestion revenues, and consequently the availability of such rights congestion pricing resolve the problem of the over recovery of transmission costs.

To solve the congestion problem, several alternatives could be considered such as re-dispatching existing generators or dispatching generators outside the congested area to supply power. The latter alternative is referred to as out-of-merit dispatch. In both alternatives, congestion has costs based on differences in energy prices between locations. In a vertically integrated monopoly, congestion costs were either ignored or hidden as bundled into the transmission charges that in turn were considered as a shortcoming in previous transmission pricing schemes. It was considered a shortcoming because it did not provide a true price signal for efficient allocation of transmission resources or allocated congestion costs to transmission customers who were not causing the congestion.

### **1.9.1 Congestion Pricing Methods:**

All new restructuring proposals are taking congestion costs into account by developing appropriate approaches to measure congestion costs and allocate these costs to transmission system users in a fair way that reflects actual use of the transmission system. These approach evolved in three basic methods based on:

- 1- **Costs of out-of-merit dispatch:** This is appropriate to systems with less significant transmission congestion problems. In this approach, congestion costs are allocated to each load on the transmission system based on its load ratio share (i.e., individual load expressed as a percent of total load).
- 2- **Locational Marginal Prices (LMPs):** This technique is based the cost of supplying energy to the next increment of load at a specific location on the transmission grid. It determines the price that buyers would pay for energy in a competitive market at specific locations and measures congestion costs by considering the difference in LMPsb etween two locations. In this approach LMPsa re calculated at all nodes of the transmission system based on bids provided to the PX.
- 3- **Usage charges of inter-zonal lines:** In this approach, the ISO region is divided into congestion zones based on the historical behavior of constrained transmission paths. Violations of transmission lines between zones (inter-zonal lines) are severe while in the congestion zone transmission constraints are small.

All transmission users who use the inter-zonal pay usage charges. These charges will be determined from bids submitted voluntarily by market participants to decrease or increase (adjust) power generation. Adjustment bids reflect a participant's willingness to increase or decrease power generation at a specified cost. An example of this approach is the case of California.

### **1.9.2 Transmission Rights:**

These rights are used to guarantee an efficient use of transmission system capacity and to allocate transmission capacity to users who value it the most. These rights are tradable rights referred to as the right to use transmission capacity and represent a claim on the physical usage of the transmission system. Moreover, these rights enable utilities to purchase existing transmission rights more cheaply than expanding the system, thereby avoiding unneeded investments. Efficient usage of the transmission can be improved by willingness to offer capacity reservations to those who value them more.

Another form of these rights is the concept of financial rights (some times called Fixed Transmission Right), which is equivalent to the physical rights. This form is proposed because it is easier for trading and less costly because the usage of a transmission system need not be tied to ownership

rights. A financial right is defined for two points on the transmission grid: injection and withdrawal points.

### **1.10 MANAGEMENT OF INTER-ZONAL/INTRAZONAL CONGESTION**

Transmission network plays a major role in the open access restructured power market. It is perceived that phase-shifters and tap transformers play vital preventive and corrective roles in congestion management. These control devices help the ISO mitigate congestion without re-dispatching generation away from preferred schedules. In this market, transmission congestion problems could be handled more easily when an inter-zonal/intra-zonal scheme is implemented.

Existing approaches to manage congestion are based on issuing orders by the ISO to various parties to re-schedule their contracts, redispatch generators, cancel some of the contracts that will lead to congestion, use various control devices, or shed loads. Other solutions are based on finding new contracts that re-direct flows on congested lines. Phase shifters, tap transformers and FACTS devices may play an important role in a restructured environment where line flows can be controlled to relieve congestion and real power losses can be minimized.

#### **1.10.1 Solution Procedure:**

Once the ISO receives preferred schedules from the PX, it performs contingency analysis by identifying the worst contingency for modeling in the congestion management. To rank the severity of different contingencies, the ISO may use a Performance Index (PI) to list and rank different contingencies. PI is a scalar function of the network variables such as voltage magnitude, real and reactive power flows. PI has essentially two functions, namely, differentiation between critical and non-critical outages, and prediction of relative severity of critical outages. There are available criteria in the literature to decide how many cases on the contingency list ought to be chosen for additional studies. A few critical contingencies at the top of the list will have the major impact on system security and should be used by the ISO during congestion management.

Economically, these price-quantity values represent what each SC is willing to pay to or receive from the ISO to remove congestion. Each schedule coordinator may trade transactions with others before submitting preferred schedules to the ISO; these parties may trade power again when preferred schedules are returned to them for revision. The preferred schedules submitted to the ISO by SC and PX are optimal schedules determined by the market clearing price, and schedules submitted by schedule coordinators are basically bilateral contracts that take into consideration the benefits of contracted parties. In this process, adjustment bids (incremental and decremental) represent the economic reformation on which the ISO will base its decisions to relieve congestion. Adjustment bids include suggested deviations from preferred loads and generation schedules provided by SCs. At each bus, ranges of power deviations along with deviations in price are submitted to the ISO. Incremental bids may be different from decremental bids for adjusting the preferred schedule.

The ISO uses incremental/decremental (inc/dec) bids to relieve congestion. Since inter-zonal congestion is more frequent than intrazonal congestion with system-wide effects, the ISO first solves interzonal congestion while ignoring intra-zonal constraints. In the inter-zonal congestion management, primary controls are zonal real power generation and loads at both ends of congested inter-zonal lines. Instead of changing preferred schedules in these zones, the ISO starts by adjusting generations and loads at buses directly connected or in proximity of these inter-zonal lines. If these controls do not accomplish the task, it is perceived that other controls away from these lines will probably not be able to mitigate inter-zonal congestion either. Then the ISO looks for other control devices (such as phase shifters, tap transformers and FACT devices) close to inter-zonal lines, however, this option requires an AC-OPF model for inter-zonal congestion management.

If no congestion is detected in any zone or on inter-zonal lines, then the submitted preferred schedules are accepted as final real time schedules.

#### **1.10.2 Formulation of Inter-Zonal Congestion Subproblem:**



The objective of the inter-zonal subproblem is represented by a modified dc load flow for adjusting preferred schedules, where the ISO minimizes the net cost of re-dispatch as determined by the SC's submitted incremental/decremental price bids. In this case, the objective is equivalent to the net power generation cost used in a conventional OPF.

For each deviation from the associated preferred schedule, a price function is provided, i.e., adjusting a generation (inc/dec) at a certain point may have a different price than that of other generators. Also, adjusting a load (dec) may present a price different from that generation or other load. These prices may represent a linear or nonlinear function of deviations, and price coefficients associated with deviations from preferred schedules reflect the SCs desire to be economically compensated for any increase or reduction in their preferred schedules. If a SC does not provide the ISO with inc/dec bids, the ISO will use inc/dec bids of other SCs for congestion management, and the SC who did not submit inc/dec bids would be automatically forced to pay congestion charges calculated according to other inc/dec bids. The formulation of this subproblem is given as follows:

### **Objective**

- Modified dc power flow to adjust preferred schedules
- Minimize the net cost of re-dispatch as determined by incremental/decremental price bids
- Objective is equivalent to the net power generation cost used in a conventional OPF

### **Control variables**

- SC's power generation in all congestion zones. For each generator a set of generation quantities with associated adjustments for incremental/decremental bids are submitted by SCs
- SCs' curtailable (adjustable) loads. For each load, a set of load quantities with associated adjustments for decremental bids are submitted by SCs. These adjustments are implicit bids for transmission across congested lines

### **Constraints**

- Limits on control variables
- Nodal active power flow balance equations
- Inter-zonal line flow inequality constraints
- Market separation between SCs

**1.10.3 Formulation of Intra-Zonal Congestion Subproblem:** At each congested zone, the ISO will use a modified AC-OPF to adjust preferred schedules. The main goal is to minimize the absolute MW of re-dispatch by taking into account the net cost of redispatch as determined by the SC's submitted incremental/decremental price bids. This objective is equivalent to the MW security re-dispatch with incremental and decremental cost-based MW weighting factors to ensure that less expensive generators are incremented first and more expensive generators are decremented first during the adjustment process. For loads, most expensive loads will be decremented first.

In each zone, congestion management is performed separately while inter-zonal constraints are preserved. The formulation may assume that loads in each zone (at each bus) can contribute to the congestion relief. any generator or load at any zonal bus is not involved in congestion management and would not submit inc/dec bids, then its minimum and maximum limits are set to preferred schedule values. The formulation of this subproblem is given as follows:

### **Objective**

- Modified AC-OPF To adjust preferred schedules
- Minimize the MW re-dispatch by taking into account the net cost of re-dispatch as determined by the SC's submitted incremental/decremental price bids
- The objective is equivalent to the MW security re-dispatch with incremental/decremental cost-based weighting factors to ensure that less expensive generators are incremented first and more

expensive generators are decremented first during the adjustment process. For loads, the most expensive ones will be decremented first.

### **Control variables**

- SCs' power generation in congested zones. For each generator a set of generation quantities with associated adjustments for incremental/decremental bids are submitted by SCs
- SCs' curtailable (adjustable) loads in the congested zone. For each load, a set of load quantities with associated adjustments for decremental bids are submitted by SCs
- Reactive power controls including:
  1. Bus voltages
  2. Reactive power injection
  3. Phase shifters
  4. Tap-transformers

### **Constraints**

- Limits on control variables
- Nodal active and reactive power flow balance equations
- Intra-zonal MVA, MW, and MVAR line flow limits (inequality constraints)
- Active power flow inequality constraints of inter-zonal lines connected to the congested zone
- Voltage limits
- Stability limits
- Contingency imposed limits

Equality constraints represent the net injection of real and reactive power at each bus in the zone. Inequality constraints reflect real power flows between buses, and stability and thermal limits define line limits. If the MVA flow limit on lines is of interest, then the MVA in equality constraint is included.

The effect of phase-shifters and tap-transformers may be seen as injections of active power and reactive power at two ends of a line between nodes where phase-shifters and tap transformers are connected. Phase-shifters and tap-transformers could also be included in the formulation by modifying the network admittance matrix.

During the intra-zonal congestion management inter-zonal line flows to the zone under study are modeled as constant loads or generations (depending on the direction of flows in inter-zonal lines) buses connected to inter-zonal lines. This modeling has two advantages:

- (1) It disregards inter-zonal line constraints that should be added to intra zonal constraints, and
- (2) It cancels interactions between inter-zonal and intra-zonal congestion subproblems while solving the intra-zonal congestion subproblem. The schedules which will be adjusted in the intra-zonal subproblems are the schedules obtained from the inter-zonal congestion subproblem.

In the intra-zonal congestion management the incremental cost coefficient of a generator at a certain node in a zone is the same as the incremental bid price. The decremental cost coefficient of a generator at a certain node in a zone is anti-symmetric with the decremental bid price with respect to the average of decremental bids in that zone. This assumption is for economical consideration, where less expensive generators would be incremented first to relieve congestion, and more expensive generators would be decremented first when generation reduction is needed.

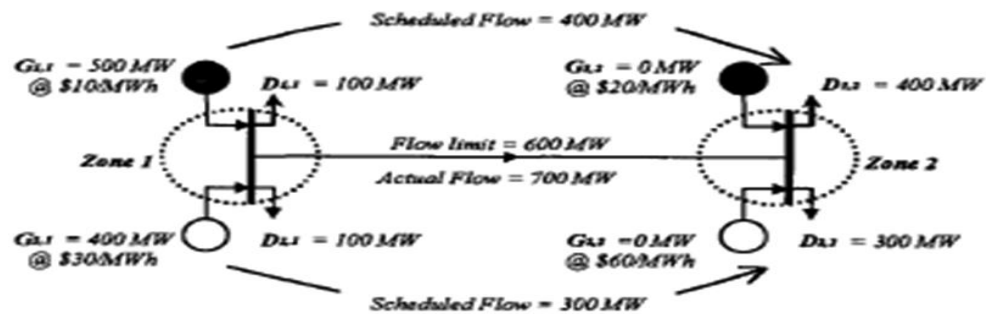
For example, if we have two generators with decremental price bids of \$10/MWh for generator  $G_A$  and \$16/MWh for generator  $G_B$  then the average decremental price bid is  $(10+16)/2=\$13/\text{MWh}$ . The decremental cost coefficients of these generators are \$16/MWh for  $G_A$  (or  $2\times 13-10$ ) and \$10/MWh (or  $2\times 13-16$ ) for  $G_B$ . The same argument is made for load reduction where more expensive loads in a zone are adjusted (decremented) first, where load increment is not considered. For the case that we have more than one provider at each bus or more than one demand, in other words, we have more than one SC at

one bus, we would index different providers at different locations in each zone. For that reason, we set three different indices in our formulation that would refer to SC, zone and bus.

### Example 1.9

#### (a) Inter-zonal congestion management

Figure 1.23 shows a simple 2-bus, 2-zone system in a certain hour, with two scheduling coordinators (SCs), where  $G_{1,1}$ ,  $G_{1,2}$ , refer to generation of SC<sub>1</sub> in zone 1 (bus 1) and zone 2 (bus 2), respectively, and  $D_{1,1}$  and  $D_{1,2}$  refer to load of SC<sub>1</sub> in zone 1 and zone 2, respectively. Also,  $G_{2,1}$ ,  $G_{2,2}$ , refer to generation of SC<sub>2</sub> in zone 1 and zone 2, respectively, and  $D_{2,1}$  and  $D_{2,2}$  refer to load of SC<sub>2</sub> in zone 1 and zone 2, respectively. The figure shows the preferred (initial) schedules of both SCs. Submitted incremental and decremental bids are given next to each generation in this figure. Incremental/decremental bids of any SC represent its implicit bids for congested paths.



**Figure 1.23 Preferred Schedules (before Congestion Management) of Example 1.19**

As shown from Figure 1.23, preferred schedules would result in a 100 MW violation on the inter-zonal line between zone 1 and zone 2. SC<sub>1</sub> produces 500 MW in zone 1, where 100 MW goes to its demand in this zone, and the rest (400 MW) crosses the inter-zonal line. Also, SC<sub>2</sub> produces 400 MW in zone 1, where 100 MW goes to its demand in this zone, and the rest (300 MW) crosses the inter-zonal line. For both SCs the flow from 1 to 2 would be 700 MW.

SC<sub>1</sub> places an implicit bid of \$10/MWh, and SC<sub>2</sub> places an implicit bid of \$30/MWh for the congested line between the two zones. Since the bid of SC<sub>2</sub> is higher than that of SC<sub>1</sub>, usage of the congested path will be allocated to SC<sub>2</sub> first, then to SC<sub>1</sub>. That means scheduled flow of 300 MW of SC<sub>2</sub> will not be altered, while the scheduled flow of SC<sub>1</sub> will be decreased until the line limit is not violated, i.e., scheduled flow of 400 MW for SC<sub>1</sub> will be decreased to 300 MW to make the actual total flow of both SCs equal to 600 MW. The solution after this step is shown in Figure 1.24. To make the required decrease in the inter-zonal path,  $G_{1,1}$  reduces its output from 500 MW to 400 MW, and  $G_{1,2}$  increases its output from 0 MW to 100 MW. After this step, note that  $G_{1,1} + G_{1,2} = D_{1,1} + D_{1,2}$  and  $G_{2,1} + G_{2,2} = D_{2,1} + D_{2,2}$ . This is a separation of markets, i.e. an increase (a decrease) in a certain SC's portfolio is compensated by a decrease (an increase) from the same SC.

**Congestion charges:**

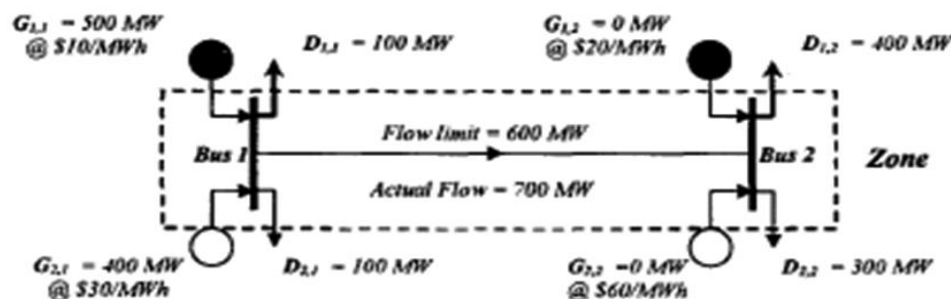
SC<sub>1</sub> is the marginal user of the congested inter-zonal line. Therefore, SC<sub>1</sub> sets the price of the congested line at \$10/MWh. The congestion charges for one hour are calculated as follows:

$$\begin{aligned} SC1 \text{ pays: } 300 \text{ MWh} \times \$10/\text{MWh} &= \$3,000 \\ SC2 \text{ pays: } 300 \text{ MWh} \times \$10/\text{MWh} &= \$3,000 \\ \text{Total} &= \$6,000 \end{aligned}$$

The ISO receives \$6,000 congestion charges from both SCs and then allocates the money to the transmission owner(s) and/or transmission right holder(s) on the path.

**(b) Intra-zonal congestion management:**

Let's assume that the 2-bus system shown in Figure 1.25 represents a certain congestion zone, and the values shown in the figure represent the schedules after the inter-zonal congestion management. We notice that the intra-zonal line connecting buses 1 and 2 is congested.



**Figure 1.25** Schedules inside a Zone before Intra-zonal Congestion Management

The generator that has the highest decremental bid at bus 1 is G<sub>2,1</sub>, so this generator will be decremented first. Also, the generator that has the lowest decremental bid at bus 2 is G<sub>1,2</sub>, so this generator will be incremented first. We need to decrease the flow in the intra-zonal line by 100 MW, so G<sub>2,1</sub> is decreased by 100 MW and G<sub>1,2</sub> is increased by 100 MW. The solution is shown in Figure 1.26.

**Intra-zonal congestion settlement:**

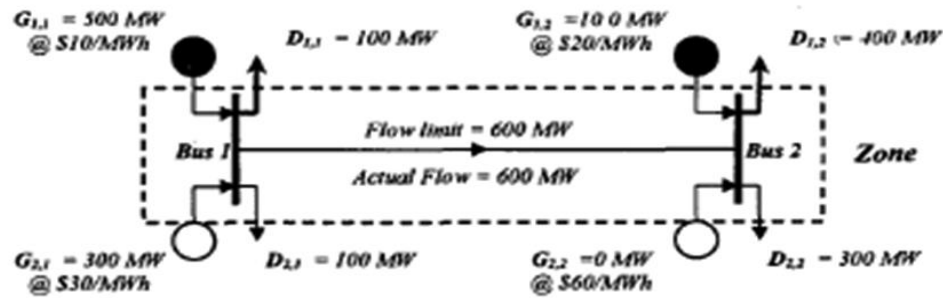
G<sub>2,1</sub> which belongs to SC<sub>2</sub> decreased its output by 100 MW.  
Payment by SC<sub>2</sub> to the ISO for G<sub>2,1</sub> is

$$100 \text{ MWh} \times \$30/\text{MWh} = \$3,000$$

G<sub>1,2</sub> which belongs to SC<sub>1</sub> increased its output by 100 MW.  
Payment by ISO to SC<sub>1</sub> for G<sub>1,2</sub> is

$$100 \text{ MWh} \times \$20/\text{MWh} = \$2,000$$





**Figure 1.26 Schedules inside a Zone after Intra-zonal Congestion Management**

Total balance of the ISO = \$3,000 - \$2,000 = \$1,000 (The ISO has a net of \$1,000). The balance (1,000) is located as a zonal uplift to SCs according to their load in and exports from the zone:

SC<sub>1</sub> gets :  $\$1,000 \times 500 \text{ MW} / 900 \text{ MW} = \$555.56$

SC<sub>2</sub> gets :  $\$1,000 \times 400 \text{ MW} / 900 \text{ MW} = \$444.44$

## 2.1 INTRODUCTION:

The demand for electricity could vary significantly according to the time of day. Electricity demand is generally higher during day-time hours, known as the peak period and lower during night-time hours, known as the off-peak period.

## 2.2 ELECTRICITY PRICE VOLATILITY:

In the restructured electric power industry, it is common to read or hear expressions such as "the volatility of electricity prices has been high during the period of January and February", "the PX market will create an environment with volatile pricing - very low at times of low demand, high at times of high demand, and very high at times of high demand and limited supply", "annualized volatility of on-peak prices", "annualized volatility of off-peak prices"; "electricity markets are highly volatile", and similar expressions which point out the volatility of electricity market. This section provides a detailed explanation of volatility and its impact on electricity pricing. In addition, the section gives a brief mathematical background on volatility, shows some examples and discusses the main motive forces for causing volatility in electricity markets.

The fact that the cost of generating electricity is based mainly on the cost and the availability of the fuel used in generation does not change by switching from regulated monopoly to open access restructured markets. Utilities used to average their fluctuating hourly costs of electricity (which were based on economic dispatch) and come up with a single cost-based rate, and users on the other side were mandated to accept this rate. Some large customers were buying electricity on an hourly-based price. In a restructured environment, hourly prices are expected to swing as they used to, with a main difference that a competitor should be competing with a large number of other competitors. In this environment, competitors bid into the market, not necessarily based on their costs but on anticipated price that takes into account movements of other competitors, market situation and supply-demand condition. This behavior would cause increased price volatility and motivates customers to take proper actions.

In order to reduce price volatility in the energy market, a trading system may allow customers to sell electric energy back through the trading system in certain hours. In the following, we further discuss the factors which could contribute to volatility in electricity markets.

### 2.2.1 Factors in Volatility:

Every electricity market is expected to have variable price patterns while proceeding from one stage to the next. This procession could be due to many factors such as entry of new players to the market, destructive gaming, bidding behavior, and availability of generation units and transmission components. As time passes, the market could correct itself to reach a final phase where prices would be predictable to a large extent and adequate rules could be implemented in modeling the whole marketing process.

During transient stages of restructuring in Britain, the electricity market experienced less price volatility due to the existence of what is called *vesting contracts*, but market participants suffered considerably from increasing uplift costs associated with ancillary services. As time passed, market rules were modified to correct the market behavior and stabilize prices. The Nordic Power Exchange (Nord Pool) has large price volatility because of its dependence on hydroelectric generation, which is in turn dependent on weather conditions, whether it is dry or rainy.

Various factors resulting in volatility include:

- **Load Uncertainty:** The power generation required to meet the load is directly correlated with weather conditions, which are sometimes unpredictable. Due to unexpected temperature changes, especially from low to high, the actual load could be at times very different from the forecasted load. If the weather forecast is uncertain, the load forecast could be uncertain.
- **Fuel Prices:** The fuel used by generating units to produce electricity is a volatile commodity with its price depending on market conditions such as demand-supply convergence conditions, transportation, storage expenses and other factors. Fossil fuel, hydro, nuclear and unconventional sources of energy could be used in generating electricity, with different costs, which are reflected on electricity prices. When marginal generating units use a certain type of fuel with fluctuating price, the electricity price could fluctuate as well.
- **Irregularity in Hydro-Electricity Production:** In some regions, hydro-electricity is produced rather inexpensively in certain times of the year due to the availability of water resources; in the remaining times of the year, thermal units are used when water quantity is reduced, which could impact electricity prices.
- **Unplanned Outages:** The imbalance between the supply and demand could cause large fluctuations in prices: When supply is less than demand or when demand is changing rapidly, price spikes could arise and when this is accompanied by a generation outage at peak hours, price spikes could be very high.
- **Constrained Transmission (Congestion):** When transmission capability is insufficient to withstand scheduled flows, the price of electricity on the load side of a congested path could be increasingly volatile and uncertain because smaller low-cost generation cannot be transmitted to loads during hours when transmission congestion exists.
- **Market Power:** Exercising market power by electricity market participants could manipulate prices and cause price volatility. In California, the PX monitors market operations, and trading rules would be altered to prevent market manipulation when these practices could arise. Market participants may use financial contracts to hedge price volatility risks. At the same time that the volatility in prices could cause large losses, it could also cause large profits if predicted earlier.
- **Market Participant:** Market participants themselves may cause price volatility in one of two ways: either by misrepresenting the actual amount of their loads or by performing gaming practices. In the first type, participants either under-schedule or over-schedule their loads. Both cases would require a response from the ISO in the imbalance energy market. Under-scheduled load may significantly change the price of energy in the imbalance market when reserves are inadequate.

### 2.2.2 Measuring Volatility:

Historical volatility is defined as the annualized standard deviation of percent changes in futures prices over a specific period. It is an indication of past volatility in the marketplace. In historical volatility, a financial variable's volatility is directly estimated from recent historical data for the

variable's value. Historical volatility gives an indication of how volatile the variable has been in the recent past for which historical data is tracked. Implied volatility is a measurement of the market's expected price range of the underlying commodity futures based on market-traded options premiums<sup>3</sup>. Implied volatility is a timely measure, which reflects the market's perceptions today. Implied volatility can be biased, especially if they are based upon options that are thinly traded.

We use *standard deviation* ( $\sigma$ ), which measures the uncertainty or dispersal of a random variable. When a financial variable (random variable) such as electric energy price is described as highly volatile, it means that it has a high standard deviation. In other words, standard deviation is a measure of the volatility of a random variable such as spot price. Figure 7.2 illustrates how standard deviation would measure the high volatility (Figure 7.2.a) and low volatility (Figure 7.2.b). Probability distribution functions in both cases are given in Figures 7.2.c and 7.2.d.

As shown in Figure 7.2, standard deviation for a specific range of a random variable X is a measure of the width of the probability distribution of the variable X. Standard deviation (i.e., square root of variance) is a measure of risk. On the other hand, the variance (expected value of squared deviations from the mean) is a measure of the dispersion of a probability distribution. As we all know,

Standard deviation =

$$\sigma = \sqrt{\text{Expected value of } [X^2] - (\text{Expected value of } [X])^2}$$

$$= \sqrt{E[X^2] - (E[X])^2}$$

$$\text{Variance} = \sigma^2 = E[X^2] - (E[X])^2$$

$$\text{Volatility}^5 = v = \sigma / \sqrt{t} ; t=1/252^6$$

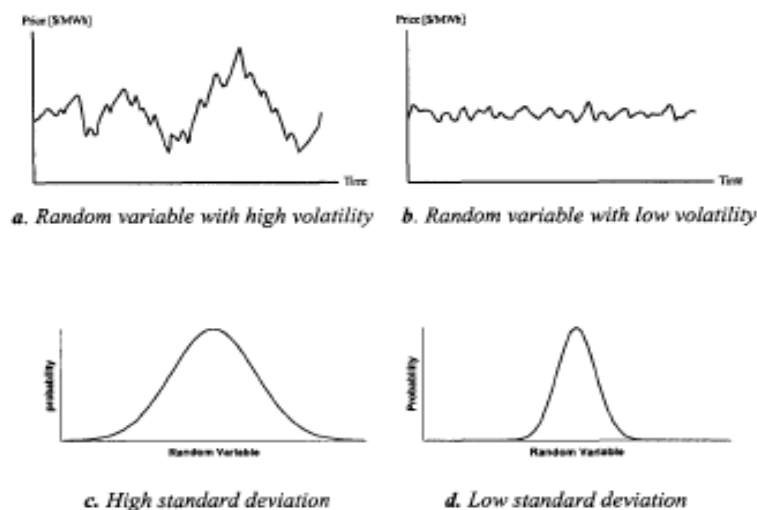


Figure 7.2 Volatility-Standard Deviation Relations

## 2.3 ELECTRICITY PRICE INDEXES:

To analyze price volatility, Dow Jones (D J) price indexes are used. Dow Jones publishes volume-weighted price indexes for the following locations (as was available on January 13, 2001):

- **California Oregon Border:** The Dow Jones California Oregon Border (D J-COB) Electricity Index is the weighted average price electric energy traded at the California-Oregon and Nevada-Oregon Borders, quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **Palo Verde:** The Dow Jones Palo Verde (DJ-PV) Electricity Index the weighted average price of electric energy traded at Palo Verde and West Wing, Arizona, quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **PJM Sellers' Choice:** The Dow Jones Pennsylvania-New Jersey- Maryland (DJ-PJM) Sellers' Choice Electricity Index is the weighted average price of electric energy traded for delivery in the Pennsylvania, New Jersey, Maryland market, quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **PJM Western Hub:** The Dow Jones Western Hub Electricity Index is the weighted average price of electric energy traded at PJM Western Hub quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **Mid-Columbia:** The Dow Jones Mid-Columbia Electricity Index is the weighted average price of electric energy traded for delivery at Mid-Columbia quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **Four corners (4C):** The Dow Jones Four Corners (DJ-4C) Electricity Index is the weighted average price of electricity traded for delivery at Four Corners, Ship rock and San Juan, New Mexico, quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **NP-15:** The Dow Jones NP-15 Electricity Index is the weighted average price of electric energy traded for delivery at NP- 15 quoted in dollars per megawatt hour. Volume in megawatt hours.
- **SP-15:** The Dow Jones SP-15 Electricity Index is the weighted average price of electricity traded for delivery at SP-15 quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **Cinergy:** The Dow Jones Cinergy Electricity Index is the weighted average price of electricity traded into the Cinergy Control Area quoted in dollars per megawatt hour. Volume is in megawatt hours.
- **Mead/Market P lace:** The Dow Jones Mead/Market Place Electricity Index is average price of electric energy traded for delivery at Mead, Market Place, McCullough and Eldorado quoted in dollars per megawatt hour. Volume is in megawatt hours.

DJ on-peak price index for COB may be compared with the on-peak PX day-ahead prices. Likewise, DJ on-peak price index for the Palo Verde (PV) may be compared with the on-peak PX day-ahead prices. Analyzing different trade operations, some specific factors are compared such as average prices, volumes of trades and volatility. The analysis can give an indication of price stability during a time period, trends in prices either declining or inclining and comparisons of volumes of trade between different locations. For example, in California, a comparison could be made for volumes traded on PX, COB and PV.

**2.3.1 Basis Risk:** The difference between the electricity spot price and the price of the nearest futures contract for the electricity at any given time is called basis. Basis risk represents the uncertainty as to whether the cash-futures spread will widen between the times a hedge position is implemented and liquidated.

## **2.4 CHALLENGES OF ELECTRICITY PRICING:**

**2.4.1 Pricing Models:** One of the major problems facing market participants, especially hedgers, in restructured electricity markets in the U.S. is the problem of large errors caused by using unsophisticated versions of the Black-Scholes<sup>10</sup> model to price physical power options. This model was originally derived as a pricing model to value European securities options and futures options. In addition to other assumptions, this model assumes that the price volatility is constant and the price series is continuous.



Some alternatives to pricing physical power options have been proposed based on this model to take into consideration the nature of electricity that is different from other commodities.

Some market participants insist on utilizing pricing models other than the Black-Scholes model. It is claimed that using the Black-Scholes model to price electricity options would result in large errors due to the assumptions that this model applies to electricity without taking into account the market's special circumstances. The Black-Scholes proposes the following price dynamics.

$$\frac{dF(t,T)}{F(t,T)} = b dW(t)$$

where,

$F(t,T)$	Price at time t for future delivery of power at time T
$b$	Constant Volatility
$dW(t)$	Standard Brownian motion <sup>11</sup> (also known as Wiener Process)

When this model is used to price the hourly or daily delivery of electric power, some problems could arise. These problems are:

- (1) Customer loads are following complex daily patterns and are sensitive to weather fluctuations which implies high volatility: The classical Black-Scholes model assumes a constant volatility, does not take into account the weather impact on volatility over the period of the option and does not discriminate between on-peak and off-peak conditions.
- (2) Electricity is a non-storable commodity: The short-term supply is largely affected by physical system dynamics such as generation and transmission outages that would result in large price spikes. The Black-Scholes model assumes smooth price changes under these circumstances.
- (3) Generating units could be forced out in unplanned manner during peak-demand summer months. Unplanned outages cause electricity prices to increase dramatically in the market due to the fact that more expensive units will be needed to serve the load.

In a nutshell, any frame work to price physical power options should take some factors into consideration. These factors include the physical nature of electricity, generation availability, dynamic volatility, transmission limitations and changeable load.

## 2.4.2 Reliable Forward Curves:

A forward curve of electricity presents a set of forward prices for electricity, i.e. it determines a set of current market prices for the sale of electricity at specified times in the future; the curve determines the present value of electricity to be delivered in the future. For other commodities that have been traded for a long time, forward curves are readily established, but for electricity in a restructured environment, much of the appropriate market information is not yet available due to the short experience. The challenge is to construct and use forward curves based on limited available data.

Forward curves in electricity markets work as bench mark or index of value. When the curve shows higher future prices, current values of production facilities and purchase agreements will increase. On the other hand, a decreasing forward curve means that the value of existing sales agreements and a utility's customer base are decreasing. In the next section, we will elaborate on forward curves.

Constructing forward curves should take the risk explicitly into account: incorporate estimates of market uncertainty in ways that will be most useful for decision-making and integrate forward curves into participants' own analytical models. To build a forward curve for midterm prices, futures and options prices should be analyzed coupled with the probabilistic system modeling. Even though the load growth and fuel price shifts affect long-term electricity prices considerably, long-term electricity prices are driven by performance improvements such as improvements in new generation technology. For long-term prices, market data provide little guidance in constructing forward curves and building the curve is mainly based on the probabilistic system modeling, asset investment and retirement analysis.

## 2.5 CONSTRUCTION OF FORWARD PRICE CURVES:

**2.5.1 Time Frame for Price Curves:** Constructing a forward curve depends mainly on a time frame, which may be for a few months (short-term), a few years (medium-term) or over several years (long-term). In short-term, the price of electricity changes mainly with changes in weather conditions, supply outages, and interregional power flows. In short-term, guidance is offered by historical spot price data coupled with deterministic system modeling. Load growth, shifts in fuel price, and customer response to retail price changes would determine medium-term price fluctuations.

### 2.5.2 Types of Forward Price Curves:

As was mentioned in the preceding section, one of the major challenges facing market participants is the lack of reliable long-dated forward prices. In restructured power markets, suppliers are competing to reach end-use customers with the lowest possible price that would guarantee profits. Winning a customer's contract is generally based on pricing strategies that would take into account electricity market trends and the information on the true cost of serving customers. The forward price of electricity is the key in pricing retail and wholesale electricity. Forward curves represent a good starting criterion to price electricity and, if utilized with experience in knowing variations in customer characteristics and supply/demands situations, they produce hedging strategies for different market participants such as suppliers, marketers, independent power suppliers and others. In this section, we highlight this topic which is very important in restructured electricity markets and the resources on this topic are very rare.

- Forward curves take on three behaviors: Backwardation, Contango, and a combination of the two.
- **Backwardation:** It is a market situation in which futures prices are lower in each succeeding delivery month. In other words, backwardation refers to markets where shorter-dated contracts are traded at a higher price than that of longer-dated contracts. Backwardation is also called the inverted market, and it is expressed by plotting the price variation with time as shown in Figure 7.8, where electricity price curve slopes downwards as time increases. Backwardation gives a forward/spot market relationship in which the forward price is lower than the spot price. The cause of backwardation in electricity markets is that it is necessary for forward prices to trend upward towards the expected spot price in order to attract speculators (buyers) to enter into trades with hedgers (sellers). The opposite of backwardation is contango.

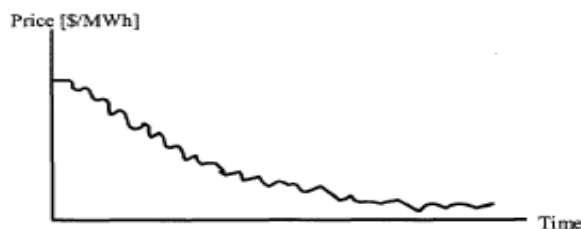


Figure 7.8 Illustration of Backwardation (Inverted Market)

- **Contango:** Opposite to the case of backwardation, contango is a term often used to refer to electricity markets where shorter-dated contracts traded at a lower price than longer-dated contracts in futures markets. When a market situation exists such that prices are higher in the succeeding delivery months than in the nearest delivery month, we say contango exists. It is expressed by plotting the prices of contracts against time, where electricity price curve slopes upwards as time increases as shown in Figure 7.9. Contango gives a forward/spot market relationship in which the forward price is greater than the spot price. Often, the forward price exceeds the spot price by approximately the net cost to carry/finance the spot electricity or security until the settlement date of the forward contract.

- **Combination:** Figure 7.10 shows a combination of the two previously mentioned behaviors of forward curves. This is an example of a situation when the forward curve takes a backwardation form in the short-term part of the curve and a combination of two in the long-term part of the curve. The behavior of the curve depends on expectations regarding the supply/demand balance in the market in addition to other seasonal factors that drive prices. In the following, we will discuss the forecasting process for the short-term price of electricity.

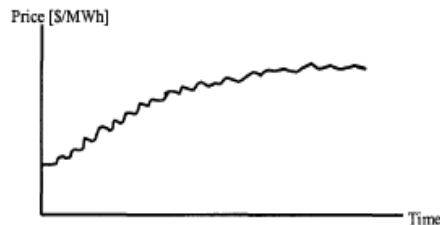


Figure 7.9 Illustration of Contango

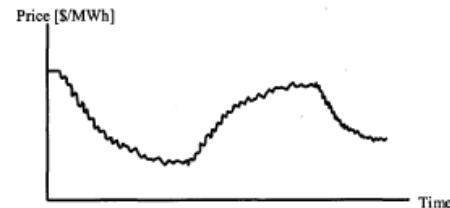


Figure 7.10 A Forward Curve Combines Backwardation and Contango

## 2.6 SHORT-TERM PRICE FORECASTING:

There are many physical factors that would impact short-term electricity price. In practice, it would be impossible to include all these factors in price forecasting, because either the factors are unknown or the related data are unavailable. The sensitivity analysis is a good way of selecting the prominent factors in price forecasting. Given a factor, if the price is insensitive to this factor, we could claim that the factor is not impacting the price and could be ignored with minute error in price forecasting.

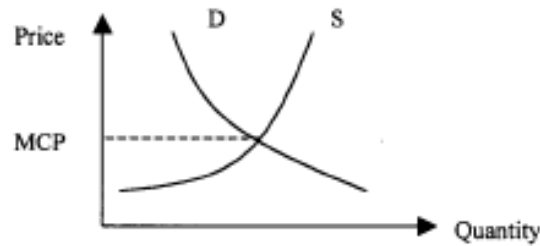
**2.6.1 Factors Impacting Electricity Price:.** An analysis of price movements presents a conceptual understanding of how factors could affect the price. For simplicity, we only discuss variations of spot price, or market clearing price (MCP) in this section.

After an auctioneer (ISO or PX) receives supply and demand bids, aggregates the supply bids into a supply curve (S) and aggregates the demand bids into a demand curve (D). The intersection (S) and (D) represents the MCP, as is illustrated in Figure 7.11.

According to this figure, we would present the following discussions.

### (1) Basic Analysis of Price Movements

- Case B1: S curve is shifted upward: MCP increases and quantity decreases.
- Case B2: S curve is shifted downward: MCP decreases and quantity increases.
- Case B3: D curve is shifted upward: MCP increases and quantity increases.
- Case B4: D curve is shifted downward: MCP decreases and quantity decreases.
- Case B5: S curve is shifted to the left: MCP increases and quantity decreases.
- Case B6: S curve is shifted to the right: MCP decreases and quantity increases.
- Case B7: D curve is shifted to left: MCP decreases and quantity decreases.
- Case B8: D curve is shifted to right: MCP increases and quantity increases.



*Figure 7.11 Calculation of MCP*

## (2) Actual Cases Pertaining To The Above Price Movements

- Case A1: Supplier would decrease the price. This is case B2.
- Case A2: Demand would increase the price. This is case B3.
- Case A3: A generator would be force-outaged (or a bid is withdrawn). This is case B5.
- Case A4: A new supplier would enter the market. This is case B6.
- Case A5: A generator would be restored. This is case B6.
- Case A6: A new demand would enter the market. This is case B8.
- Case A7: Gas (or oil) price would decrease. Suppliers would then decrease their prices. So, it is case B2.
- Case A8: Gas (or oil) price would increase. Suppliers would then increase their price. So, it is case B1.

It is vital to perform the above seemingly simple analysis, as it would exhibit the variation of price in practical markets. For example, we would learn, from the above analysis, that the price of gas (or oil) could affect MCP.

### 2.6.2 Forecasting Methods:

- **Simulation Method:** Usually the analysis of price volatility is based on the probability distribution for each of a series of key drivers. The users can determine the distribution of input variables using historical data. For example providers could use a beta distribution, which requires the estimation of the maximum, minimum and the most likely value of input variables. To capture the effects of uncertainty, samples are drawn from the distribution of the input variables using Monte Carlo methods and a scenario is created. For each scenario the tool is used to simulate the market prices. Running a sufficient number of scenarios then produces a stable distribution of long-term market prices.
- **Artificial Neural Network Method:** The artificial neural network method has received more attention in the field of forecasting because of its clear model, easy implementation and good performance. The method was applied before to load forecasting in electric power systems. Here, we use the MATLAB for training the artificial neural network in short-term price forecasting, which provides a very powerful tool for analyzing factors that could impact electricity prices.

### 2.6.3 ANALYZING FORECASTING ERRORS:



Let  $V_a$  be the actual value and  $V_f$  the forecast value. Then, Percentage Error (PE) is defined as

$$PE = (V_f - V_a) / V_a \times 100\% \quad (7.6)$$

and the Absolute Percentage Error (APE) is

$$APE = |PE| \quad (7.7)$$

then, the Mean Absolute Percentage Error (MAPE) is given as

$$MAPE = \frac{1}{N} \sum_{i=1}^N APE_i \quad (7.8)$$

MAPE is widely used to evaluate the performance of load forecasting. However in price, forecasting, MAPE is not a reasonable criterion as it may lead to inaccurate representation. The problem with this MAPE is that if the actual value is large and the forecasted value is small, then APE will be close to 100%. In addition, if the actual value is small, APE could be very large if the difference between actual and forecasted values is small. For instance, when the actual value is zero APE could reach infinity if the forecast is not zero. So, there is a problem with using APE for price forecasting. It should be noted that this problem does arise in load forecasting, since actual values are rather large, while price could be very small or even zero.

**Alternate Definition of MAPE:** One proposed alternative is as follows. First we define the average value for a variable  $V$ :

$$\bar{V} = \frac{1}{N} \sum_{i=1}^N V_a \quad (7.9)$$

Then, we redefine PE, APE and MAPE as follows:

Percentage Error (PE):

$$PE = (V_f - V_a) / \bar{V} \times 100\% \quad (7.10)$$

Absolute Percentage Error (APE):

$$APE = |PE| \quad (7.11)$$

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{1}{N} \sum_{i=1}^N APE_i \quad (7.12)$$

The point here is that we would use the average value as the basis to avoid the volatility problem.

**2.6.4 Practical Data Study:** In this section, we use artificial neural networks to study price forecasting based on the practical data. We will study the impact of data pre-processing, quantities of training vectors, quantities of impacting factors, and adaptive forecasting on price forecasting. We will also compare the artificial neural network method with alternative methods. The new definition of MAPE is illustrated with practical data and its advantages are discussed.

#### 2.6.4.1 Impact of Data Pre-Processing:

The improvement in training MAPE is due to the disappearance of price spikes (excluded or limited). Consequently, without price spikes, network training can find a more general mapping between input and output. Thus, testing MAPE will also be improved.

Since price spikes are the indicative of abnormality in the system, we do not intend to delete them from the training process. Hence, we adhere to the option of limiting the magnitude of spikes, rather than eliminating them totally.

#### **2.6.4.2 Impact of Training Vectors:**

At first, by introducing more training vectors, we present a more diverse set of training samples, which would result in a more general input-output mapping. Thus, the forecasting performance, measured by the testing MAPE, would improve. However, as we keep increasing the number of training vectors, the diversity of training samples would no longer expand and the additional training would not improve the forecasting results. Thus, the forecasting performance would remain flat. We should point out that by further increasing the number of training vectors, the artificial neural network could be over trained. In other words, the artificial neural network would have to adjust its weights to accommodate the input-output mapping of a large number of training vectors that may not be similar to the testing data to a large extent. Thus, the forecasting performance could get worse with further increasing the number of training vectors.

**2.6.4.3 Impact of Adaptive Forecasting:** We can either use the fixed training weights or upgrade the weights frequently and adaptively according to the test results. We refer to the latter case as our adaptive forecasting method. Studying the profile of price curves, we would expect that the adaptive modification of network weights would provide a better forecast.

### **2.7 CONCLUSIONS:**

The demand for price transparency increased ever since the restructuring process began and the number of participants and marketing operations increased. This is due to the need to enhance the financial stability of electricity markets, which is in turn due to changes in strategies or approaches to buy and sell electricity, which is completely different from the traditional methods under the regulated monopoly. Add to that is mergers of new financial tools and entry of non-electricity participants in electricity markets. All these facts motivated participants to demand efficient tools for price discovery in order to hedge their risks and survive in a competitive market. In this chapter, we have reviewed some basic concepts in electricity price forecasting, such as price calculation and price volatility. Because of its importance, we also discussed the issue on factors impacting electricity price forecasting, including time factors, load factors, historical price factor, etc. We used the artificial neural network method to study the relationship between these factors and price. We proposed a more reasonable definition on MAPE to avoid the demerits of traditional methods on measuring forecasting in the context of electricity price forecasting. Practical data study showed that a good data pre-processing was helpful, i.e., using too many training vectors or considering too many factors is not good for price forecasting. Practical data study also showed adaptive forecasting could improve forecasting accuracy. We concluded that the artificial neural network method is a good tool for price forecasting as compared to other methods in terms of accuracy as well as convenience.